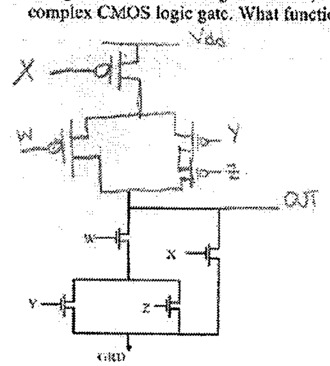
ECE PHD PRELIMINARY EXAMINATION

SOLUTIONS – SPRING 2007 EXAM

Ph.D. Preliminary Examination Solutions
Spring 2007
Problem 1 (Core: CompE-ECE2030) Code Number:__________

ECE2030 (CORE) Prelim Problem

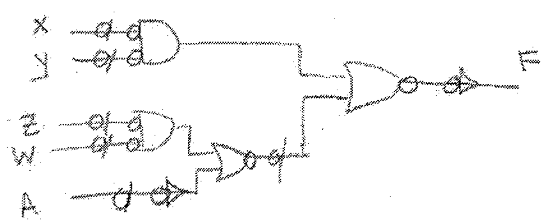
a. Using the nFET network given below, draw the corresponding pFET network to complete this complex CMOS logic gate. What function does this circuit implement?



$OUT = \overline{X}(\overline{W} + \overline{Z}\,\overline{Y})$

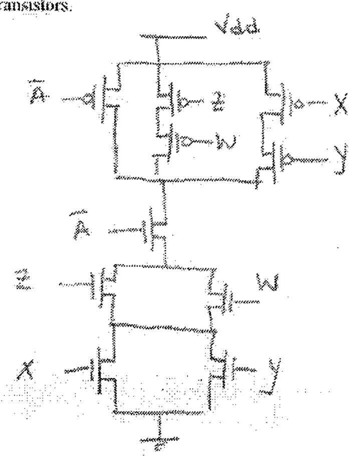
b. Show the LOGIC schematic for the following Boolean function. Please show the implementation (preferably with mixed logic notation) using NOR gates and INVERTERS only.

$F = \overline{X}\,\overline{Y} + \overline{\overline{Z}\,\overline{W} + A}$



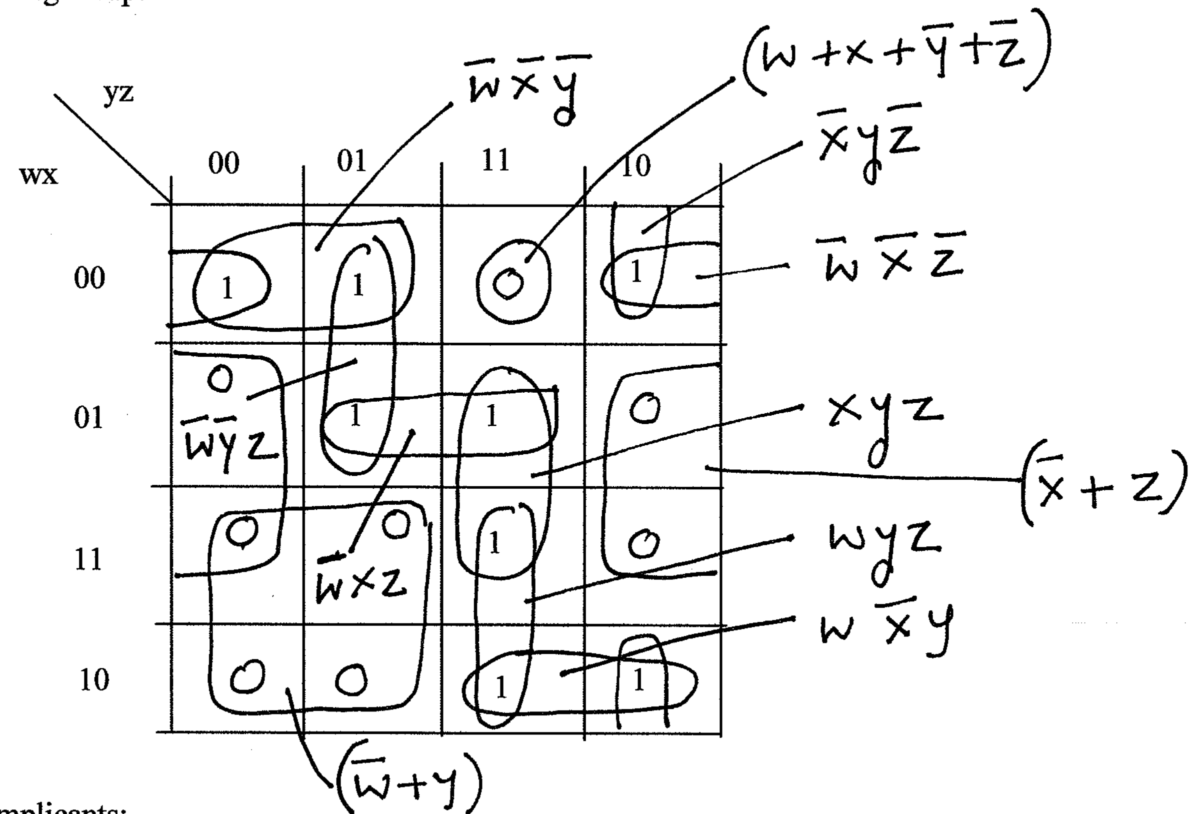
c. Show the TRANSISTOR schematic for a complex CMOS gate that implements the following function with a minimum number of transistors.

$OUT = \overline{X}\,\overline{Y} + \overline{Z}\,\overline{W} + A$



Problem 2 (Core: CompE-ECE2030) **Code Number:__________**

(a) List all the *prime implicants* of the boolean function f(w,x,y,z) given by the following Karnaugh map:



Prime implicants:

$\bar{w}\bar{x}\bar{y}$, $\bar{x}y\bar{z}$, $\bar{w}\bar{x}\bar{z}$, xyz, wyz, $w\bar{x}y$, $\bar{w}xz$ and $\bar{w}\bar{y}z$

(b) List all the *essential prime implicants* of the boolean function f( ), above.

Essential prime implicants:

There are no essential PIs.

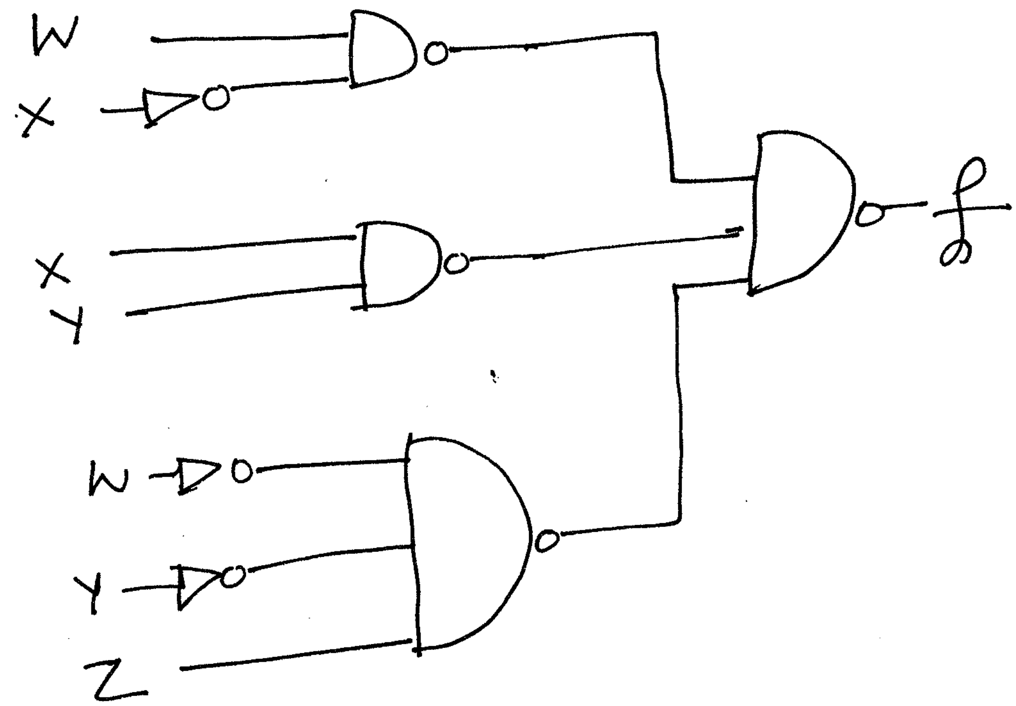
(c) Give a minimal *sum of products* form expression for the function f( ) below:

f(wxyz) = $\bar{w}\bar{x}\bar{y} + \bar{w}xz + wyz + \bar{x}y\bar{z}$

(d) Give a minimal *product of sums* expression for the same function f( ) below:

f(wxyz) = $(\bar{w}+y)\cdot(\bar{x}+z)\cdot(w+x+\bar{y}+\bar{z})$

(e) Show an implementation of the function $f(wxyz) = w\bar{x} + xy + \overline{wy}z$ using a *minimum* number of inverters and NAND gates only. You may use two or three input NAND gates in your design.



Problem 3 (Core: CompE-ECE3055)   Code Number:_________

The following RISC assembly language program is executed on a 32-bit MIPS processor. Fill in the register values that will be present, after execution of this program. A summary of MIPS instructions is included at the bottom of the page – for anyone unfamiliar with the MIPS instruction set. Prior to execution of the program, memory location 0x01000 contains 0x30552031. ***Note:*** 0x indicates hexadecimal and all answers must be in hexadecimal, default is decimal in the MIPS assembly language source file. A MIPS memory word or register contains 32-bits. Use XXXXXXXX for an undefined value.

```
          LW      $3, 0x01000     R3=30552031
          SLL     $4, $3, 4       R4=05520310
          ADD     $2, $3, $4      R2=35A72341
          AND     $3, $4, $3      R3=00500010
          LUI     $5, 0x3055      R5=30550000
          ORI     $5, $5, 38      R5=30550026
          SUB     $6, $4, $3      R6=05020300
          BNE     $3, $6, LABEL1
          ADDI    $6, $0, -20
LABEL1:   SW      $6, 0x01000
```

After execution of the MIPS code sequence above,

R2 = 0x 35A72341 *(in hexadecimal)*

R3 = 0x 00500010 *(in hexadecimal)*

R4 = 0x 05520310 *(in hexadecimal)*

R5 = 0x 30550026 *(in hexadecimal)*

Memory Location 0x01000 contains: 0x 05020300 *(in hexadecimal)*

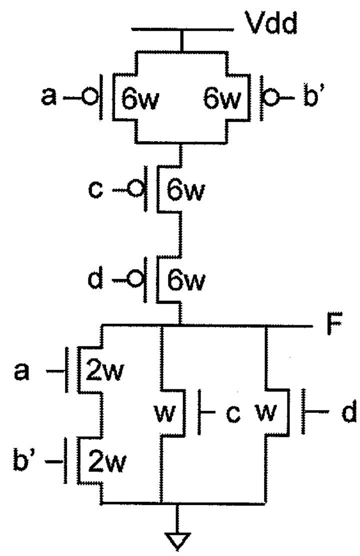
The MIPS processor contains thirty-two 32-bit registers, $0 through $31. $0 always contains a zero. By default, all arithmetic operations use two's complement arithmetic. Assume no branch delay slot is present.

MIPS Instruction			*Meaning*
ADD	Rd, Rs, Rt	-	Rd = Rs + Rt (R – register ($))
AND	Rd, Rs, Rt	-	Rd = Rs bitwise logical AND Rt (R – register ($))
ORI	Rd, Rs, *Immed*	-	Rd = Rs bitwise logical OR *Immediate* value
LUI	Rd, *Immed*	-	Rd = 16-bit *Immediate* value high 16-bits, 0's low 16-bits
BNE	Rs, Rt, *address*	-	Branch to *address*, only if Rs not equal to Rt
LW	Rd, *address*	-	LOAD - Rd gets contents of memory at *address*
SLL	Rd, Rs, *count*	-	Shift left logical (*use 0 fill*) by *count* bits
SUB	Rd, Rs, Rt	-	Rd = Rs - Rt
SW	Rd, *address*	-	STORE - memory at *address* gets contents of Rd
XOR	Rd, Rs, Rt	-	Rd = Rs bitwise logical XOR Rt

Problem 4 (Core: CompE-ECE3060) **Code Number:_________**

Solutions:

a) $F = \overline{a\bar{b}} \cdot \overline{c + d} = (\bar{a} + b)\bar{c}\bar{d}$



b) pull-up: $3 \cdot 2R/3 \cdot 10Cinv = 20\tau$. pull-down: $2 \cdot R/3 \cdot 10Cinv = 20/3\tau$. Thus, the worst-case delay is 20τ.

c) sizing is shown in the figure above

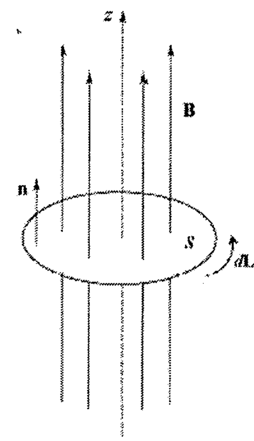
Problem 5 (Core: E&M-ECE3025) Code Number:_________

Prelim Problem, ECE 3025

One of Maxwell's equations is Faraday's law of induction, expressed below in point and integral forms: The left-hand integral is taken over a contour, C, that forms the closed boundary of surface S. The unit vector, $\mathbf{n}$, is normal to that surface. The direction of the contour integral is defined according to the right-hand convention, in which the thumb of the right hand is directed along the normal, $\mathbf{n}$, while the fingers of the right hand curl in the direction of the contour integral (or $d\mathbf{L}$).

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{L} = -\frac{d}{dt} \int\int_S \mathbf{B} \cdot \mathbf{n}\, da$$



a. Suppose a spatially-uniform, but time-varying magnetic field, $\mathbf{B}(t)$, penetrates a circular surface, S, in the xy plane, with the magnetic field oriented in the positive z direction. $\mathbf{B}$ is known to *decrease* with time. Obtain the correct orientation of the induced electric field, and express this direction as an appropriate unit vector in cylindrical coordinates.

Ans: As $\mathbf{B}$ decreases with time ($dB/dt < 0$), and is in the same direction as the normal to the surface ($\mathbf{B} \cdot \mathbf{n} > 0$), the integral equation indicates that $\mathbf{E} \cdot d\mathbf{L}$ will be net positive. $\mathbf{E}$ is thus in the direction of $d\mathbf{L}$, or $+\mathbf{a}_\phi$. oriented

b. Describe the modifications that could be made to the given $\mathbf{B}$ field that would lead to the same answer as in part *a*.

Ans: Reversing the direction of $\mathbf{B}$ *and* making it *increase* with time would lead to the same electric field magnitude and direction as in part *a*.

c. Under the conditions of part *a*, write an expression for the magnitude of the net induced *voltage* around a circular loop of radius a, with the loop in the xy plane. Express your result in terms of $B(t)$ and other pertinent quantities.

Ans: With $\mathbf{B}$ spatially-uniform, and with area πa^2 within the loop, the induced voltage (or emf) is

$$\left| \oint \mathbf{E} \cdot d\mathbf{L} \right| = \pi a^2 \left| \frac{dB}{dt} \right|$$

d. Under the conditions of part *a*, write an expression for the magnitude and direction of the induced *electric field* at radius a. Express your result in terms of $B(t)$ and other pertinent quantities.

Ans: $\mathbf{E}$ will be constant around the loop. Having already found the voltage, we would therefore have

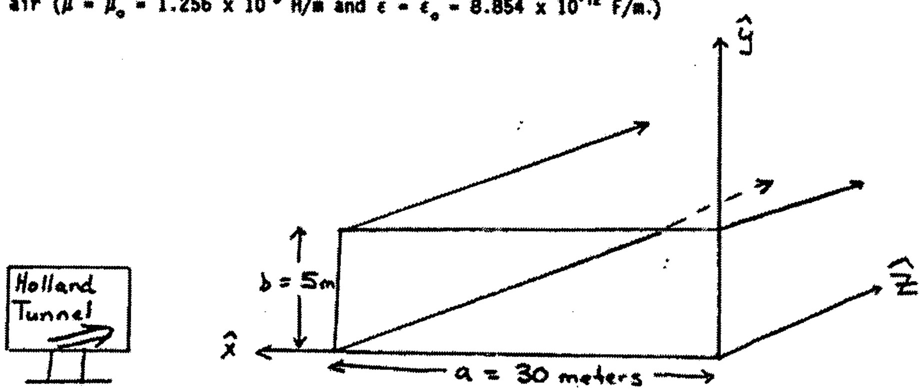
$$\mathbf{E} = \frac{\text{emf}}{2\pi a} = \frac{a}{2} \left| \frac{dB}{dt} \right| \mathbf{a}_\phi$$

Problem 6 (Core: E&M-ECE3065) **Code Number:__________**

Solution

Problem 6 (Core EM – ECE 3065) **Code Number:_______**

The Holland Tunnel has a rectangular cross-section with dimensions shown below. Because of the water which permeates the walls of the tunnel, the walls can be considered as nearly perfect conductors, and the tunnel is filled with lossless air ($\mu = \mu_o = 1.256 \times 10^{-6}$ H/m and $\epsilon = \epsilon_o = 8.854 \times 10^{-12}$ F/m.)



a) What is the cut-off frequency for the dominant waveguide mode propagating inside the tunnel?

Dominant mode is TE_{10}

f_c = 5×10^6 Hz where $f_c = \frac{1}{\sqrt{\mu\epsilon}} / 2a$

b) What waveguide mode will have the next highest cut-off frequency?

TE_{20} (name of mode) Since waveguide is so wide, next highest mode is TE_{20}.

c) Will signals from local AM radio stations (assume f = 1000 kHz = 10^6 Hz) propagate inside the tunnel? (Yes or No, and Why?)

Yes **(No)** (Circle one)

Why? $f < f_c$, ∴ Waveguide is below cut-off

d) Assuming a 6 MHz (6×10^6 Hz) transmitter is placed at the entrance to the tunnel, what will be the wavelength of propagating waves inside the tunnel?

λ_g = 90.45 meters

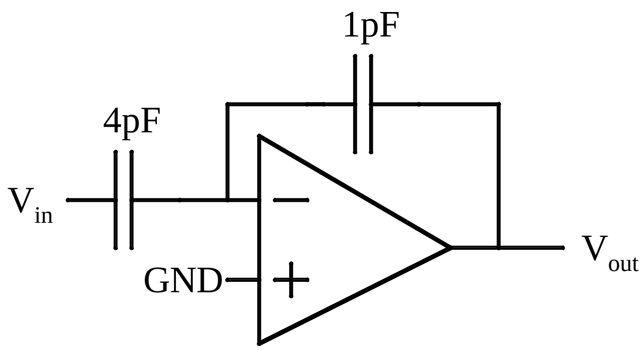
A waveguide is a "Wave-stretcher":

$$\lambda_{fs} = \frac{3\times10^8 \text{ m/sec}}{f} = 50 \text{ meters}$$

$$\lambda_g = \frac{\lambda_{\text{free-space}}}{\sqrt{1-(f_c/f)^2}} = \frac{50 \text{ meters}}{\sqrt{1-(5/6)^2}} =$$

Problem 7 (Core: EDA-ECE2040) **Code Number: ___________**

Consider the circuit below. Assume the op-amp is an ideal op-amp. You must write your answers in the line (where a line is given) below to receive any credit.



Solution: What is the input resistance / impedance looking into the input terminals of an ideal op-amp?

__Infinite Resistance___

For a DC input voltage of 0V applied to the input, the DC output voltage is 2V. What is the cause of the 2V DC output voltage?

Charge Stored at the node connected to the 1pF, 4pF and – op amp terminal.

For the rest of the problem, assume the statement to be true for a DC input voltage of 0V applied to the input, the DC output voltage is 2V. Solve for the dc gain and the bandpass gain.

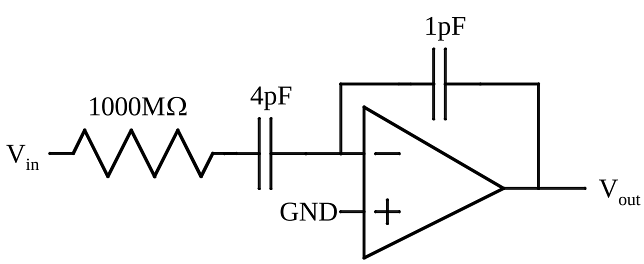
Dc gain = ______-4__________ (impedance at the – terminal is infinite even with the feedback)
(did accept an answer of 0 if justified)

Bandpass gain = ___-4_________

s (4pF) V_{in} = s (1pF) ($-V_{out}$); therefore $V_{out} / V_{in} = -4$.

Rigorously, this solution holds upto s=0, therefore gain is -4, but at DC, the gain is undetermined by this method of analysis. A more rigorous analysis even at DC gives a gain of -4. One could argue that in a physical system that charge might not be held infinitely, so the gain at DC might be 0. Of course, a physical op-amp would also not have infinite gain, etc, so not a strong argument.

For the circuit below, solve for the dc gain, the bandpass gain, and the corner frequency for the resulting circuit. Assume the op-amp is an ideal op-amp. You must write your answers in the line below to receive any credit. For this circuit, when a dc input of 0V is applied, the dc output voltage is 0V.



Analyzing this circuit, we get

(V_{in} - V)/ 1000MΩ = s (4pF) V = s (1pF) ($-V_{out}$); therefore V_{out} / V = -4.

V(s) / Vin(s) = 1 / (1 + s (4pF)(1000MΩ)) = 1 / (1 + s (4ms))

Dc gain = _____-4______

Bandpass gain = _____-4____

Corner frequency = __1 / (2 π 4ms) _ = _39.8 Hz (~ 40Hz)_________

Problem 8 (Core: EDA-ECE3050/3041) **Code Number: ___________**

Assume you have a MOSFET transistor with all four terminals available, where one of the terminals is a gate, one is a source, one is a drain, and one is a well terminal, but none of the terminals were labeled. You know that none of the terminals are tied together. Your job is to determine an experimental procedure to identify all four terminals using typical laboratory benchtop equipment (i.e. voltage sources, voltage meters, current meters), as well as identify whether the device is an nFET or a pFET transistor.

<u>**Solution:**</u> There are a few strategies for solving this problem; we will present the easiest approach, but all approaches take a similar form of their solution.

A MOSFET is a 4 terminal device. The one terminal that never has current flowing into it is the gate terminal, because the gate is a capacitor to the silicon substrate. All other three terminals can have a forward biased pn junction in particular conditions, which we will utilize a bit later on in this problem.

Therefore, the first problem is developing a set of experiments to check for a terminal that never pulls any current. Therefore, take all of the pairwise set of terminals, while keeping the other two terminals floating (and therefore forcing the resulting current through those terminals equal to 0), and perform a voltage sweep while measuring the current flowing between the selected two terminals. After finishing these six measurements, there are 3 measurements that yield effectively zero current with one terminal in common. That common terminal is the gate for the device.

Next question is finding the bulk terminal from the remaining three devices. The other two terminals are the source terminal and the drain terminal, which for a fabricated MOSFET are identical. We know that electrically we see a p-n junction device from source or drain to the bulk potential; therefore if we look at the sweep data from the above experiments that did not include the gate terminal, we should be able to figure out which terminal is the bulk terminal. Two other experiments will look like a pn junction with a forward bias and a reverse bias region; there would be no current from the other terminal, since it would be disconnected. These two experiments will have the bulk terminal in common. One experiment will be inconclusive, because it will be looking at the current between source and drain; depending upon the location of the surface potential one might get a wide range of measured current when sweeping the voltage, but would not get a pn junction response that matches the other two devices. One could get an asymmetric response, but since the source and drain are identical, we would expect nearly identical responses from two of the curves. Therefore, we select the common terminal between these two sweeps, to give us the well terminal. The other two terminals are the source and drain terminals.

Finally, further looking at the two sweeps to determine the bulk terminal, we can determine whether we have an nFET or a pFET device. Remember we have a p-n junction between bulk and source-drain junction; an nFET would have p-type substrate (and n+ type source-drain junctions), where a pFET would have n-type substrate (and p+ type source-drain junctions). Therefore, if we had exponentially rising current dependance when the bulk terminal was higher than the source-drain junctions, we have an nFET, and if we had exponentially rising current dependance when the bulk terminal was lower than the source-drain junctions, we have a pFET.

Problem 9 (Core: Power-ECE3070) Code Number:_________

a) The core loss conductance is given by,

$$G_c = \frac{P_{oc}}{V_{oc}^2} = \frac{100}{2200^2} = 20.66\,\mu\text{S}$$

The exciting impedance is,

$$Y_{oc} = \frac{I_{oc}}{V_{oc}} = \frac{2}{2200} = 0.909\,\text{mS}$$

The magnetizing susceptance is then,

$$B_m = \sqrt{Y_{oc}^2 - G_c^2} = 0.908\,\text{mS}$$

b) Referring these values to the high voltage side,

$$B_m^{HV} = B_m^{LV}\left(\frac{N_{LV}}{N_{HV}}\right)^2 = 0.908\text{E-3}\left(\frac{2200}{11{,}000}\right)^2 = 0.3635\,\mu\text{S}$$

$$G_c^{HV} = G_c^{LV}\left(\frac{N_{LV}}{N_{HV}}\right)^2 = 20.66\text{E-6}\left(\frac{2200}{11{,}000}\right)^2 = 8.264\text{E-8S}$$

c) The power would be unchanged, but the current would be divided by the turns ratio.

$$I_{oc}^{HV} = I_{oc}^{LV}\left(\frac{2200}{11000}\right) = 2\left(\frac{2200}{11000}\right) = 0.4\,\text{A}$$

d) The rating of the autotransformer is the rated primary current times the rated input voltage. As an autotransformer the primary (input) current is the rated current of the 2200 V winding. Therefore, the rating is;

$$13{,}200\left(\frac{100{,}000}{2200}\right) = 600\,\text{kVA}$$

Problem 10 (Core: Power-ECE3070) **Code Number:__________**

SOLUTION

PER PHASE ROTOR IMPEDANCE REFERRED TO THE STATOR

$Z_f = R_f + jX_f = \left(\frac{R_R'}{s} + jX_R'\right) \| jX_M = 5.41 + j3.11\ \Omega$ (FOR $s = 0.02$)

STATOR INPUT IMPEDANCE IS

$Z_{IN} = R_1 + jX_1 + Z_f = 5.70 + j3.61 = 6.75\angle 32.3°\ \Omega$

NOMINAL LINE-TO-NEUTRAL VOLTAGE (AND CURRENT)

$\tilde{V}_1 = 220/\sqrt{3} = 127\ V$ $\tilde{I}_1 = \frac{\tilde{V}_1}{Z_{IN}} = \frac{127}{6.75\angle 32.3°} = 18.8\angle -32.3°\ A$

NOMINAL SPEED IS $n_s = \left(\frac{120}{p}\right) \cdot 60 = \left(\frac{120}{6}\right) 60 = 1,200\ rpm$

AND ANGULAR VELOCITY $\omega_s = \frac{4\pi f_e}{p} = 125.7\ rad/sec$

ROTOR SPEED IS $n_r = (1-s) n_s = 0.98 \cdot 1,200 = 1,176\ rpm$

$\omega_r = (1-s)\omega_s = 0.98 \cdot 125.7 = 123.2\ rad/sec$

POWER TRANSFERRED OVER THE GAP IS

$P_{gap} = n_{ph} I_1^2 R_f = 3 \cdot (18.8)^2 \cdot 5.41 = 5,740\ W$

$P_{shaft} = P_{mech} - P_{rot} = (1-s) P_{gap} - P_{rot} = 0.98 \cdot 5,740 - 403 = 5,220\ W$

THE SHAFT OUTPUT TORQUE IS

$T_{shaft} = \frac{P_{shaft}}{\omega_m} = \frac{5220}{123.2} = 42.4\ N \cdot m$

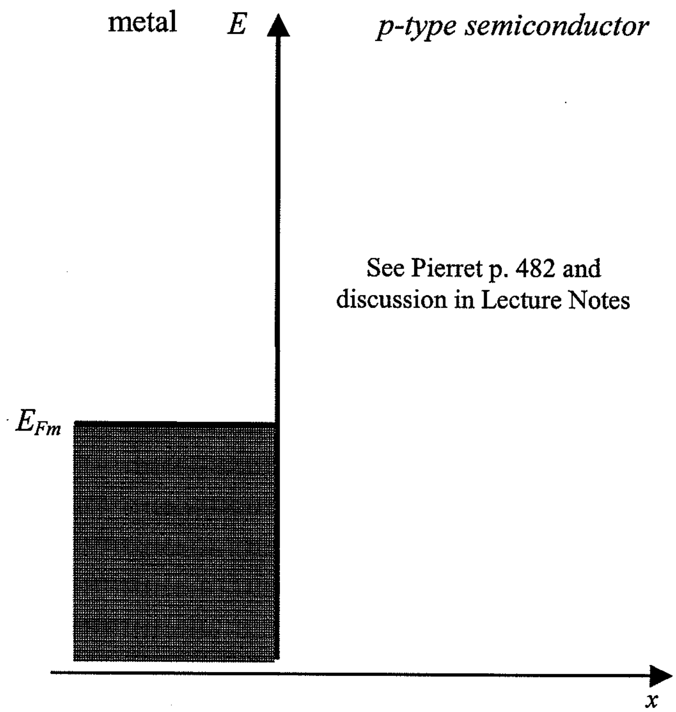
TOTAL INPUT POWER IS

$P_{IN} = n_{ph} \cdot Re\{\tilde{V}_1 \tilde{I}_1^*\} = 3 \cdot Re\{127 \cdot 18.8\angle 32.3°\} = 3 \cdot 127 \cdot 18.8 \cdot \cos(32.3°) = 6,060\ W$

THE MACHINE EFFICIENCY IS

$\eta = \frac{P_{shaft}}{P_{in}} = \frac{5,220}{6,060} = 0.861 = 86.1\ \%$

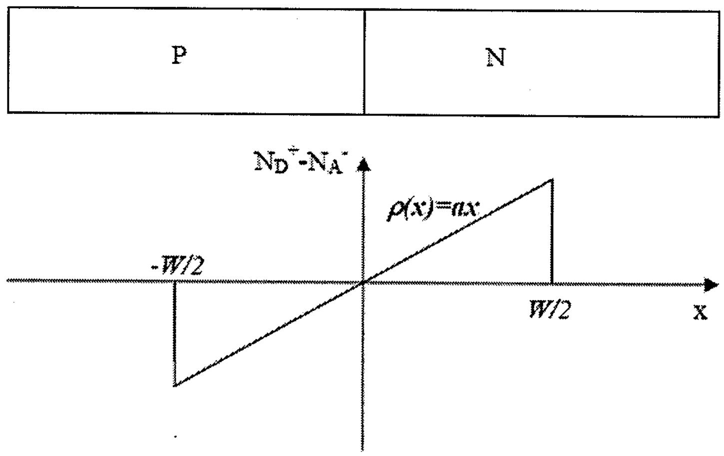
Problem 11 (Core: Microsystems-ECE3040) Code Number:_________



Problem 12 (Core: Microsystems-ECE3080) Code Number:__________

Solution:

From the depletion approximation, one may assume that the depletion width is equally divided between the N-side and P-side of the junction due to symmetric doping profile.



The Poisson's equation leads to the following derivation:

$$\nabla^2 V = -\frac{\rho(x)}{\varepsilon_s}$$

$$\rho(x) = \begin{cases} ax & -\frac{W}{2} \le x \le \frac{W}{2} \\ 0 & elsewhere \end{cases}$$

The electric field $\xi = -\nabla V$

$$\nabla \xi = \frac{\partial \xi}{\partial x} = \frac{\rho(x)}{\varepsilon_s} \quad (1-D)$$

$$\xi(x) = \int_{-W/2}^{x} \frac{qax}{\varepsilon_o \varepsilon_s} dx = \frac{qa(x^2 - \frac{W^2}{4})}{2\varepsilon_o \varepsilon_s}$$

$$V(x) = -\int_{-W/2}^{x} \xi(x) dx = \left. \frac{-qa(\frac{1}{3}x^3 - \frac{W^2}{4}x)}{2\varepsilon_o \varepsilon_s} \right|_{\frac{-W}{2}}^{x} = \frac{qa}{6\varepsilon_s \varepsilon_o}[2(\frac{W}{2})^3 + 3(\frac{W}{2})^2 x - x^3] \quad -\frac{W}{2} \le x \le \frac{W}{2}$$

$$V(W/2) = V_{bi} = \frac{-qa(\frac{1}{3}\frac{W^3}{8} - \frac{W^2}{4}\frac{W}{2})}{2\varepsilon_o \varepsilon_s} + \frac{qa(\frac{-1}{3}\frac{W^3}{8} + \frac{W^2}{4}\frac{W}{2})}{2\varepsilon_o \varepsilon_s} = \frac{qaW^3}{12\varepsilon_o \varepsilon_s}$$

$$W = (\frac{12\varepsilon_o \varepsilon_s}{qa} V_{bi})^{1/3}$$

Problem 13 (Core: DSP-ECE2025) Code Number:_________

Solution

a) Setting $y_1(t) = f^2(t)$, we have

$$\hat{y}_1(\omega) = \frac{1}{2\pi}\left(\hat{f}(\omega) * \hat{f}(\omega)\right).$$

Since $\hat{f}(\omega) = 0$ for $|\omega| > \Omega$, $(\hat{f} * \hat{f})$ will be zero outside a band twice as large; that is

$$\hat{y}_1(\omega) = 0 \quad \text{for} \quad |\omega| > 2\Omega.$$

b) The $y_d[n]$ are samples of $f^2(t)$ — the squaring operation is applied pointwise, and hence commutes with the sampling operator. The Shannon-Nyquist sampling theorem tells us that if the sample spacing T obeys

$$T \leq \frac{2\pi}{\text{BW}},$$

where BW is the bandwidth of the signal being sampled, we can reconstruct the signal perfectly using the sinc interpolator shown in the problem. Here, we have $\text{BW} = 4\Omega$, and so

$$T \leq \frac{\pi}{2\Omega}$$

will guarantee that $y_2(t) = y_1(t)$. If $T < \pi/(2\Omega)$, it is easy to find a bandlimited f such that $y_1(t) \neq y_2(t)$. Take, for example, f with

$$\hat{f}(\omega) = \begin{cases} 1 & |\omega| \leq \Omega \\ 0 & |\omega| > \Omega \end{cases}.$$

c) Using similar reasoning above, we can deduce the bandwidth of each of the terms in the polynomial: $f^2(t)$ has bandwidth 4Ω, $f^3(t)$ has bandwidth $6\Omega,\ldots, f^N(t)$ has bandwidth $2N\Omega$ (the constants $a_1, \ldots, a_N$ do not affect the bandwidth). It is easy to see that the bandwidth of the sum of each of these terms is $2N\Omega$. Thus we need

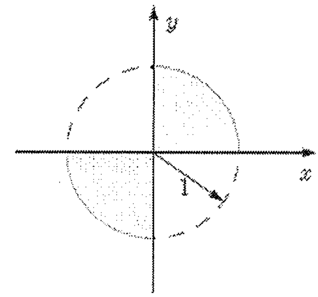
$$T \leq \frac{\pi}{N\Omega}$$

to have $y_2(t) = y_1(t)$ for all $f(t)$ bandlimited to Ω.

Problem 14 (Core: DSP-ECE3075) Code Number:_________

3075 SOLUTION

Let X and Y be random variables that are uniformly distributed over the shaded region shown to the right, so that the joint pdf for X and Y is:



$$f(x,y) = \begin{cases} K, & \text{for } (x, y) \in \text{shaded region} \\ 0, & \text{elsewhere} \end{cases}$$

where K is an appropriate constant.

(a) Are X and Y independent? ☐ ☒
Explain

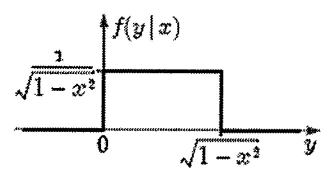
By symmetry, both X and Y are zero mean. Without computing a numerical value, we can see that X and Y are *positively* correlated: $E[XY]$ will be positive because $f(x, y)$ is nonzero only when x and y have the same sign. And correlated RV's are not independent.

(b) *Sketch* the conditional pdf $f(y \mid x)$, carefully labeling both axes.

The conditional pdf is $f(y \mid x) = f(x, y)/f(x)$, where the numerator "slices" the joint pdf along the given value of x, and the denominator scales the result so that it integrates to unity. Slicing and scaling the given joint pdf will yield one of two conditional pdf's, depending on the sign of x.

When $x > 0$, it will look like this:

When $x < 0$ it will be the same, only reflected about $y = 0$.



(c) Define a new random variable $R = \sqrt{X^2 + Y^2}$.
Find an equation for, and also carefully sketch, its pdf $f(r)$.

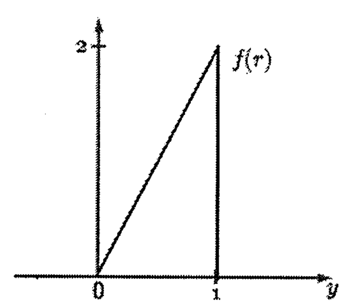
It's cdf is $$F(r) = \Pr[\sqrt{X^2+Y^2} < r] = \iint_{\substack{\text{circle of} \\ \text{radius } r}} f(x, y)\,dx\,dy$$

$$= K \cdot 0.5\pi r^2 = r^2 \qquad \text{for } 0 < r < 1.$$

Differentiating gives the pdf:

$$f(r) = \frac{d}{dr}F(r) = 2r, \qquad \text{for } 0 < r < 1,$$

as sketched here:



Problem 15 (Core: S&C-ECE3085) Code Number:__________

Solution:

Step 1:

1) $E(s) = R(s) - C(s)$

2) $\dfrac{C(s)}{R(s)} = (1+Ks)\left[\dfrac{G(s)}{1+G(s)}\right] = (1+Ks)\left[\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right] = \dfrac{\omega_n^2(1+Ks)}{s^2+2\xi\omega_n s+\omega_n^2}$

3) $C(s) = \dfrac{\omega_n^2(1+Ks)}{s^2+2\xi\omega_n s+\omega_n^2}R(s)$

Step 2:

Substituting (3) into (1)

$$E(s) = R(s) - \frac{\omega_n^2(1+Ks)}{s^2+2\xi\omega_n s+\omega_n^2}R(s)$$

$$E(s) = R(s)\left[\frac{s^2+2\xi\omega_n s-\omega_n^2 Ks}{s^2+2\xi\omega_n s+\omega_n^2}\right]$$

Since R(s) is a ramp input, $R(s) = \dfrac{1}{s^2}$

Therefore $E(s) = \dfrac{1}{s^2}\left[\dfrac{s^2+2\xi\omega_n s-\omega_n^2 Ks}{s^2+2\xi\omega_n s+\omega_n^2}\right]$

Step 3:

To find equation for steady state error:

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{s^2}\left[\frac{s^2+2\xi\omega_n s-\omega_n^2 Ks}{s^2+2\xi\omega_n s+\omega_n^2}\right] = \lim_{s\to 0}\frac{s+2\xi\omega_n-\omega_n^2 K}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{2\xi\omega_n-\omega_n^2 K}{\omega_n^2}$$

Step 4:

To eliminate steady state error:

$$e_{ss} = 0 \Rightarrow = \frac{2\xi\omega_n-\omega_n^2 K}{\omega_n^2} = 0 \Rightarrow 2\xi\omega_n - \omega_n^2 K = 0 \Rightarrow K = \frac{2\xi}{\omega_n}$$

Problem 16 (Core: S&C-ECE3085) **Code Number:__________**

Solution:

o) Nyquist plot starts for $\omega = 0$ at 3 and end for $\omega = \infty$ at 0. The RL branch must start at one of the poles (here $(1+j\sqrt{3})/2$) and approach the zero at $s = -3$ via a breakpoint somewhere on the negative real axis. The natural (for increasing k) direction is thus from right to left.

i) Trivial: Since $\omega_0 \in \Omega$, with $H(j\omega_0) = -N_0$. Recall that the Nyquist plot is nothing but $\Im H(j\omega)$ versus $\Re H(j\omega)$ as $\omega : 0 \to \infty$.

ii) This is a reformulation of the previous: $\Im H(s) = 0$ when $s = j\omega_0$. Recall that each of the Root Locus branches traces a solution, s to $H(s) = -1/k$ as $k : 0 \to \infty$. Thus the point $j\omega_0$ lies on the Root Locus since $\Im(H(j\omega_0) = 0$ and $\Im(-1/k) = 0$.

iii) First note that in the limit for $\Delta\omega \to 0$,

$$H(j(\omega_0 + \Delta\omega)) - H(j\omega_0) = H'(j\omega_0)\, j\Delta\omega.$$

Hence the Nyquist path leaves from frequency ω_0 (the crossing) at an angle given by

$$\arg\left(H'(j\omega_0)\, j\Delta\omega\right) = \arg\left(H'(j\omega_0)\right) + \frac{\pi}{2}.$$

But ϕ_N, the *directed* angle from the real axis to the Nyquist plot, is then the negative of this quantity: Thus

$$\phi_N = -\arg\left(H'(j\omega_0)\right) - \frac{\pi}{2}.$$

Likewise the (directed) angle between the RL branch and the imaginary axis follows from

$$H(j\omega_0 + \Delta s) - H(j\omega_0) = -\frac{1}{k_0 + \Delta k} + \frac{1}{k_0}$$

where corresponding to $s = j\omega_0$, we have $k_0 = 1/N_0$. But the left hand side approaches for small Δs the complex number $H'(j\omega_0)\Delta s$. For small $\Delta k > 0$, the right hand side approaches

$$-\frac{1}{k_0 + \Delta k} + \frac{1}{k_0} = -\frac{1}{k_0}\left[\frac{1}{1 + \frac{\Delta k}{k_0}} - 1\right] \to \frac{\Delta k}{k_0^2}.$$

It follows that the directed angle from the imaginary axis to the RL branch is

$$\phi_{RL} = \arg(\Delta s) - \frac{\pi}{2} = \arg\left(\frac{\Delta k}{k_0^2 H'(j\omega_0)}\right) - \frac{\pi}{2} = -\arg\left(H'(j\omega_0)\right) - \frac{\pi}{2} = \phi_N.$$

Problem 17 (Specialized: Comp Science-CS3210) Code Number:______

SOLUTION

Uses MIPS assembly and assumes semaphore is stored at memory location loc_sem.

```
acquire:    ll      r1, loc_sem
            blez    r1, acquire
            addi    r1, r1, -1
            sc      r1, loc_sem, r2
            beqz    r2, acquire
            return

release:    ll      r3, loc_sem
            addi    r3, r3, 1
            sc      r3, loc_sem, r4
            beqz    r4, release
            return
```

Problem 18 (Specialized: Software Sys- ECE3035) Code Number: _____

Prelim Problem Solutions **Computing Mechanisms**

Below is a snapshot of heap storage in byte addressed memory. Values that are pointers are denoted with a "$". The heap pointer is $6112. The heap has been allocated contiguously beginning at $6000, with no gaps between objects. Objects are word-aligned. Each allocated object has a header word containing the size of the object in bytes. The address of the object points to the word just after the header (i.e., the first word allocated for the object's data). For example, the object pointed to by $6004 has size 16 bytes, so the next allocated object in the heap is of size 8 bytes and is pointed to by $6024.

addr	value	addr	value	addr	value	addr	value	addr	value
6000	16	6032	12	6064	8	6096	12	6128	0
6004	33	6036	28	6068	4	6100	$6024	6132	0
6008	$6004	6040	24	6072	$6092	6104	16	6136	0
6012	16	6044	$6100	6076	8	6108	0	6140	0
6016	3	6048	12	6080	$6068	6112	0	6144	0
6020	8	6052	$6080	6084	0	6116	0	6148	0
6024	25	6056	16	6088	4	6120	0	6152	0
6028	52	6060	$6004	6092	$6068	6124	0	6156	0

Part A Suppose the stack holds a local variable whose value is the memory address $6052. No registers or static variables currently hold heap memory addresses. List the addresses of all objects in the heap that are *not* garbage.

Addresses of Non-Garbage Objects: **$6004, $6052, $6068, $6080, $6092**

Non-garbage objects are color-coded above (green indicates an object header containing size info in bytes and blue indicates the space allocated for the object's data). The address of each object is the address of the first word of data allocated for the object. Non-garbage objects are those reachable from the root address $6052.

Part B If an object of size 15 bytes is allocated, what address will be returned as a pointer to the newly allocated object using a first-fit allocation strategy?

Address: **$6116**

None of the objects on the free list (i.e., those that are garbage) are large enough to hold 15 bytes. So the new object is allocated at the top of the heap. Its size info goes in $6112 and $6116 is the pointer returned to the newly allocated object.

Part C If the local variable whose value is the address $6052 is popped from the stack, which addresses listed in Part A will be reclaimed by each of the following strategies? If none, write "none."

Reference Counting:	**$6052, $6080 (the rest are involved in cyclic references)**
Mark and Sweep:	**$6004, $6052, $6068, $6080, $6092**
Old-New Space (copying):	**$6004, $6052, $6068, $6080, $6092**

Problem 19 (Specialized: Telecom-ECE3076) Code Number:__________

Between hosts A in Atlanta and B in Las Vegas there are 2 routers (X,Y). The link between routers (---) is 10 Mbps. The access links (LANs, ===) are 1000 Mbps. The distance from A to B is 2500 km. A starts to send a large file using TCP, sending 1500 byte packets to B. B ACKs with 40 byte packets. There is no other traffic on this network.

A ===X---Y===B

What is the time required to transmit a 1200 Byte datagram at 10 Mbit/s?
______________**0.96**_____ ms

What is the propagation delay for the round trip in milliseconds (ms):
____________**25**__________ ms

If the router buffers are empty, what is the total round trip transmission delay (neglect processing delay)? ___ **26.15**____ ms

What is the average transport rate for a 5 kByte Window size and this RTT? _____**1.53 Mbps** ________

How big a Window size would be needed to reach 10 Mbit/s? __________**32.7 kByte**____________

What would happen if the window size was 65 kByte? ___**Limited to 10 Mbps by X-T link speed**. {Buffer delay will increase to limit A's transmission rate. If overflow occurs, A's window size will decrease.} __________

Other traffic builds up the average level in X's X-to-Y output buffer to 50 kBytes. What does this queuing delay add to the RTT? ____ **40** ____ms

What is the average transport rate for this new (total) RTT (Window still 5 kByte)?
_**0.605** _ Mbits/s

What will happen to reduce the rate A sends bits if packets start being lost? __**Window size decreases** __

Why is the output buffer delay for router Y not a concern? ___ **It's output is faster than it's input** _____

Problem 20 (Specialized: Optics-ECE4500) Code Number:__________

Solutions:

1) The electric permittivity tensor is a second-rank tensor that is diagonal in the principal axis and can be written as:

$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}$$

In a uniaxial crystal $\varepsilon_{xx} = \varepsilon_{yy}$

2) From the constitutive equations, the relationship between refractive index and electric permittivity is

$$\varepsilon = \varepsilon_0 \varepsilon_r \quad \textit{with} \quad n^2 = \varepsilon_r$$

Hence,

$$\varepsilon_{xx} = \varepsilon_0 n_o^2 = 2 \times 10^{-11} \ F/m$$
$$\varepsilon_{zz} = \varepsilon_0 n_e^2 = 1.9 \times 10^{-11} \ F/m$$

3) Index ellipsoid in x = 0 plane writes:

$$\frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

4) For s-polarized wave: $n = n_o = 1.507$

5) For p-polarized wave:

$$\frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2} = \frac{1}{n^2}$$

$\sin^2 45 = \cos^2 45 = 1/2$

$$n = \left(\frac{2 n_e^2 n_o^2}{n_e^2 + n_o^2} \right)^{1/2} = 1.487$$

Problem 21 (Specialized: Optics-ECE4501) Code Number:_________

tangential components of the field vectors at each interface. In addition to the continuity conditions at the interfaces, another important boundary condition for guided modes is that the field amplitudes be zero at infinity. A discussion of the general properties of the solutions of Equation (3.1-4) will be given later in this book.

We now proceed with the solution of Equation (3.1-4) in each segment of the dielectric structure. The propagation constant β is a very important parameter because it determines whether the field varies sinusoidally or exponentially. For confined modes, the field amplitude must fall off exponentially outside the guide structure. Consequently, the quantity $(n\omega/c)^2 - \beta^2$ in Equation (3.1-4) must be negative for $|x| > \frac{1}{2}d$. In other words, the propagation constant β of a confined mode must be such that

$$\beta > \frac{n_1\omega}{c} \tag{3.1-5}$$

where we recall that n_1 is the index of refraction of the bounding media. On the other hand, the continuity of the field requires that the magnitude of the field $E_m(x)$ attain a maximum value. The existence of a maximum requires that the Laplacian of the field be negative. In other words, the propagation constant of a confined mode must be such that

$$\beta < \frac{n_2\omega}{c} \tag{3.1-6}$$

Thus we will find confined modes whose propagation constant satisfies these conditions, Equations (3.1-5) and (3.1-6). The modes can also be classified as either TE or TM modes. The TE modes have their electric field perpendicular to the xz plane (plane of incidence, or plane of propagation) and thus have only the field components E_y, H_x, and H_z. The TM modes have the field components H_y, E_x, and E_z.

Guided TE Modes

The electric field amplitude of the guided TE modes can be written in the form

$$E_y(x, z, t) = E_m(x) \exp[i(\omega t - \beta z)] \tag{3.1-7}$$

In a manner very similar to the wavefunction of a particle in a square-well potential, the mode function $E_m(x)$ is taken as

$$E_m(x) = \begin{cases} A \sin hx + B \cos hx, & |x| < \frac{1}{2}d \\ C \exp(-qx), & x > \frac{1}{2}d \\ D \exp(qx), & x < -\frac{1}{2}d \end{cases} \tag{3.1-8}$$

where A, B, C, and D are constants, and the parameters h and q are related to the propagation constant by

$$h = \left[\left(\frac{n_2\omega}{c}\right)^2 - \beta^2\right]^{1/2}$$
$$q = \left[\beta^2 - \left(\frac{n_1\omega}{c}\right)^2\right]^{1/2} \tag{3.1-9}$$

The parameter h may be considered as the transverse component of the wavevector in the guiding layer. To be acceptable solutions, the tangential component of the electric and magnetic fields, E_y and H_z, must be continuous at the interfaces. Since $H_z = (i/\omega\mu)(\partial E_y/\partial x)$, we must match the magnitude as well as the slope of the TE mode functions $E_m(x)$ at the interfaces. This leads to

$$A \sin(\tfrac{1}{2}hd) + B \cos(\tfrac{1}{2}hd) = C \exp(-\tfrac{1}{2}qd)$$

$$hA \cos(\tfrac{1}{2}hd) - hB \sin(\tfrac{1}{2}hd) = -qC \exp(-\tfrac{1}{2}qd)$$

$$-A \sin(\tfrac{1}{2}hd) + B \cos(\tfrac{1}{2}hd) = D \exp(-\tfrac{1}{2}qd)$$

$$hA \cos(\tfrac{1}{2}hd) + hB \sin(\tfrac{1}{2}hd) = qD \exp(-\tfrac{1}{2}qd)$$

from which we obtain

$$2A \sin(\tfrac{1}{2}hd) = (C - D) \exp(-\tfrac{1}{2}qd) \tag{3.1-10}$$

$$2hA \cos(\tfrac{1}{2}hd) = -q(C - D) \exp(-\tfrac{1}{2}qd) \tag{3.1-11}$$

$$2B \cos(\tfrac{1}{2}hd) = (C + D) \exp(-\tfrac{1}{2}qd) \tag{3.1-12}$$

$$2hB \sin(\tfrac{1}{2}hd) = q(C + D) \exp(-\tfrac{1}{2}qd) \tag{3.1-13}$$

By examining the above equations, we find there are two sets of solutions.

(a) Symmetric modes ($A = 0$ and $C = D$): Equations (3.1-12) and (3.1-13) yield

$$h \tan(\tfrac{1}{2}hd) = q \quad \text{(for symmetric TE modes)} \tag{3.1-14}$$

(b) Antisymmetric modes ($B = 0$ and $C = -D$): Equations (3.1-10) and (3.1-11) give

$$h \cot(\tfrac{1}{2}hd) = -q \quad \text{(for antisymmetric TE modes)} \tag{3.1-15}$$

Note that both Equations (3.1-14) and (3.1-15) cannot be satisfied simultaneously since the elimination of q would lead to a pure imaginary h and a negative q. However, these two equations can be combined into a single equation (see Problem 3.14):

$$\tan(hd) = \frac{2hq}{h^2 - q^2} \tag{3.1-16}$$

The solutions of TE modes may thus be divided into two classes. For the first class,

$$A = 0, \quad C = D, \quad h \tan(\tfrac{1}{2}hd) = q \tag{3.1-17}$$

and for the second class,

$$B = 0, \quad C = -D, \quad h \cot(\tfrac{1}{2}hd) = -q \tag{3.1-18}$$

Note that the solutions in the first class have symmetric wavefunctions, whereas those of the second class have antisymmetric wavefunctions.

The propagation constants of the TE modes are found from a numerical or graphical solution of Equations (3.1-17) and (3.1-18), with the definition of h and q given by Equation (3.1-9). A very simple and well-known graphic solution is described here, since it clearly shows the way in which the number of TE modes depends on both the thickness d and the difference of indices of refraction. By putting $u = \tfrac{1}{2}hd$ and $v = \tfrac{1}{2}qd$, Equation (3.1-17) becomes $u \tan u = v$, with

$$u^2 + v^2 = (n_2^2 - n_1^2)\left(\frac{\omega d}{2c}\right)^2 = (n_2^2 - n_1^2)\left(\frac{\pi d}{\lambda}\right)^2 \equiv V^2 \tag{3.1-19}$$

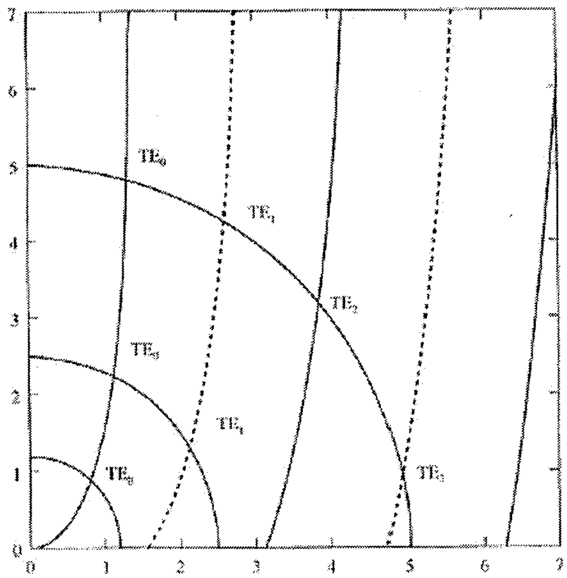


Figure 3.2 Graphic solution of Equations (3.1-17) and (3.1-18) for three values of V. Solid curves are $v = u \tan u$, and the dotted curves are $v = -u \cot u$.

Since u and v are restricted to positive values, the propagation constants may be found in this case from the intersection of both the curve $v = u \tan u$ and a circle of known radius $V = (n_2^2 - n_1^2)^{1/2}(\pi d/\lambda)$ in the first quadrant of the uv plane. A similar graphic construction for the solution of Equation (3.1-18) can be obtained by plotting $v = -u \cot u$ and the circle on the uv plane. Figure 3.2 shows such a graphic method for three values of V. For $V = 1.2$, there is only one solution—the TE_0 mode. There are two solutions (TE_0 and TE_1) when $V = 2.5$ and four solutions when $V = 5$. Note that the number of solutions depends on the value of V.

From Figure 3.2, it is clear that the number of confined TE modes depends on the magnitude of the parameter V. For V between zero and $\frac{1}{2}\pi$, there is just one TE mode of the first class. The first mode of the second class appears when the parameter V is greater than $\frac{1}{2}\pi$. As this parameter V increases, confined modes appear successively, first of one class and then of the other. Figure 3.3 plots the wavefunctions of a symmetrical slab waveguide with $n_2 = 1.6$, $n_1 = 1.5$, $d = 5$ μm, and $\lambda = 1.55$ μm. According to Equation (3.1-19), the parameter $V = 5.64$. This waveguide supports four TE modes. It is not difficult to see from Figure 3.3 that, when ordered according to the propagation constant β, the mth wavefunction has $m - 1$ nodes. We also notice that the wavefunctions are either symmetric or antisymmetric with respect to the origin $x = 0$. It follows from the discussion earlier that the wavefunctions are divided into two classes (see Equations (3.1-17) and (3.1-18)). This division is a direct consequence of the fact that the index profile $n(x)$ is symmetric about $x = 0$.

Knowledge that the solution possesses a definite symmetry sometimes simplifies the determination of the propagation constant, since we need only find the solution for positive x. Even solutions have zero slope and odd solutions have zero value at the origin $x = 0$. Thus the wavefunction of the even solutions can be written as $\cos(hx)$, whereas those of the odd solutions can be written as $\sin(hx)$. Both types of solutions decay exponentially in the region $|x| > \frac{1}{2}d$. The solutions are then obtained by matching the value and the slope at $|x| = \frac{1}{2}d$.

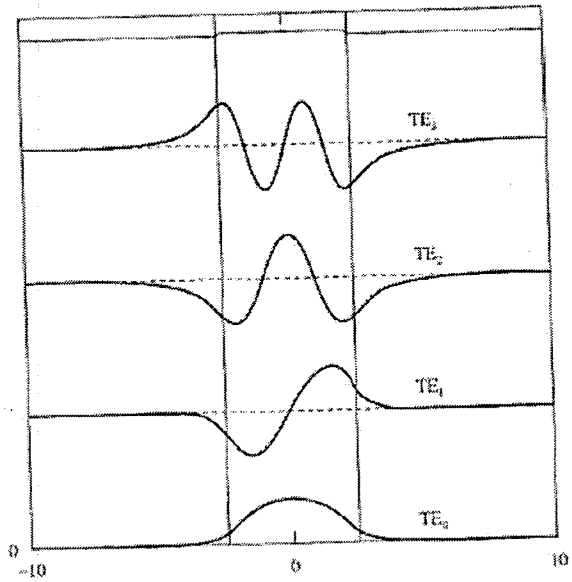


Figure 3.3 Wavefunctions of a symmetrical slab waveguide with $n_2 = 1.6$ and $n_1 = 1.5$. The thickness of the core is equal to $d = 5$ μm, and $\lambda = 1.55$ μm. The normalized propagation constants are 1.5946, 1.5785, 1.5521, and 1.5175. The fundamental mode has the largest propagation constant. The confinement factors (fraction of energy inside the core) for the modes are $\Gamma = 0.9914$, 0.9631, 0.9033 and 0.7511. The fundamental mode has the highest confinement.

For the purpose of describing and comparing the confined modes, it is convenient to define the normalized propagation constant as

$$\bar{\beta} = \frac{\beta}{\omega/c} \tag{3.1-20}$$

Such a normalized propagation is often called the effective index of refraction of the mode, n_{eff}, and is related to the phase velocity of the mode:

$$\bar{\beta} = n_{\text{eff}} = \frac{c}{v_p} \tag{3.1-21}$$

where v_p is the phase velocity of the mode, $v_p = \omega/\beta$. Thus, for confined modes, the normalized propagation constant $\bar{\beta}$ or the effective index n_{eff} is between n_2 and n_1.

Guided TM Modes

We now consider the TM modes whose magnetic field vector is perpendicular to the plane of propagation (xz plane). The derivation of the confined TM modes is similar in principle to that of the TE modes. The field amplitudes are written

$$\begin{aligned} H_y(x, z, t) &= H_m(x) \exp[i(\omega t - \beta z)] \\ E_x(x, z, t) &= \frac{i}{\omega\mu}\frac{\partial}{\partial z} H_y \\ E_z(x, z, t) &= -\frac{i}{\omega\mu}\frac{\partial}{\partial x} H_y \end{aligned} \tag{3.1-22}$$

The wavefunction $H_m(x)$ is

$$H_m(x) = \begin{cases} A \sin hx + B \cos hx, & |x| < \frac{1}{2}d \\ C \exp(-qx), & x > \frac{1}{2}d \\ D \exp(qx), & x < -\frac{1}{2}d \end{cases} \tag{3.1-23}$$

where A, B, C, and D are constants, and the parameters h and q are given by Equation (3.1-9).

The continuity of H_y and E_z at the two interfaces $x = \pm\frac{1}{2}d$ leads, in a manner similar to Equations (3.1-14) and (3.1-15), to the following eigenvalue equation:

$$\begin{aligned} h \tan(\tfrac{1}{2}hd) &= \frac{n_2^2}{n_1^2} q \quad \text{for even solutions} \\ h \cot(\tfrac{1}{2}hd) &= -\frac{n_2^2}{n_1^2} q \quad \text{for odd solutions} \end{aligned} \tag{3.1-24}$$

These two equations can also be combined into a single equation,

$$\tan(hd) = \frac{2h\bar{q}}{h^2 - \bar{q}^2} \tag{3.1-25}$$

where

$$\bar{q} = \frac{n_2^2}{n_1^2} q \tag{3.1-26}$$

Equation (3.1-24) can also be solved by using the graphic method described earlier. Figure 3.4 shows the dispersion relation (effective index n_{eff} versus normalized frequency V) of a typical symmetric waveguide.

The frequency at which $q = 0$ is a cutoff frequency. For a mode with $q = 0$, the field is no longer exponentially decaying in the cladding region and the propagation is no longer confined. Referring to Figure 3.4, we note that TE_0 and TM_0 modes have no cutoff frequency. In other words, these two modes are always confined in a symmetric waveguide. $V = \pi/2$ is

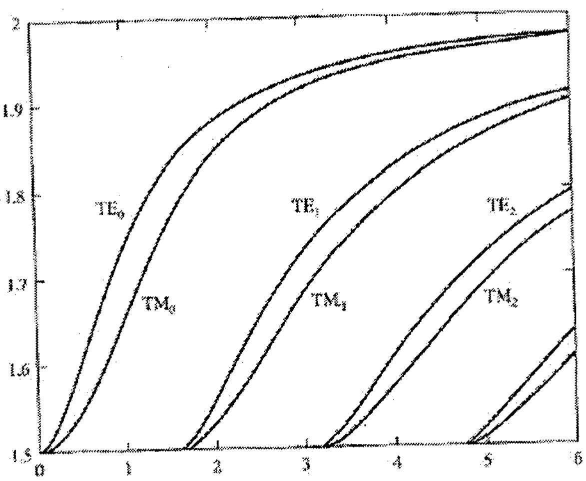


Figure 3.4 The effective index n_{eff} versus normalized frequency V of a typical symmetric waveguide with $n_1 = 1.5$ and $n_2 = 2.0$.

Problem 22 (Specialized: Microsystems-ECE4451) Code Number:_____

Solutions:

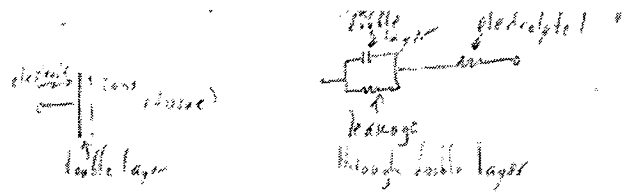
1. The minority carriers for n-type semiconductor are holes.
2. There is no light illumination at $t>0$. The time-dependent minority carrier continuity equation is

$$\frac{\partial \Delta p_n(x,t)}{\partial t} = -\frac{\Delta p_n}{\tau_p} - \mu_p E \frac{\partial \Delta p_n}{\partial x} + D_p \frac{\partial^2 \Delta p_n}{\partial x^2}$$

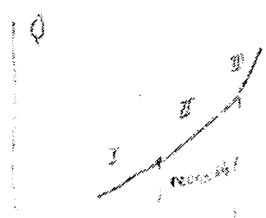
3. The drift velocity $v_d=(0.45cm-0.05cm)/(0.1ms-0.2ms)=4000$ cm/s
4. The mobility for holes is $\mu_p=v_d/E=4000/2=2000$ $cm^2/(V\text{-}s)$. From Einstein relation, $D_p=(kT/q)\mu_p=0.0259*2000=51.8$ cm^2/s

Problem 23 (Specialized: Bio Eng-ECE4781) Code Number:__________

a)

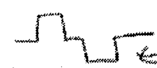


b)



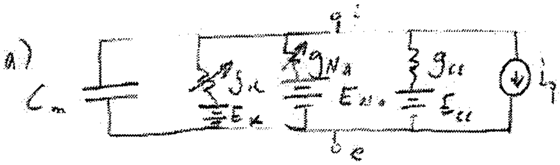
I & III: [illegible] changes on electrode [illegible]

c) large dia. but there are more small diameter which are more likely to be stimulated

d) apply a reset  to discharge the Faraday Capacitance

Problem 24 (Specialized: Bio Eng-ECE4781) Code Number:__________

a)



C_m = membrane capacitance associated with lipid bilayer as dielectric & i + e electrolytes as conductive plates

g_K = membrane potassium conductance = $g_K(V_m, t)$ ⇒ brought about by both resting and voltage-gated channels

g_{Na} = same as g_K only for Na

g_{Cl} = membrane Cl conductance ≠ $g_{Cl}(V_m, t)$ ⇒ only resting Cl channels are involved

E_K, E_{Na}, E_{Cl} = Nernst potentials for those ions

i_p = ion pump — pumps $3 Na^+$ out and $2 K^+$ in per pump cycle. This pump is ATP activated

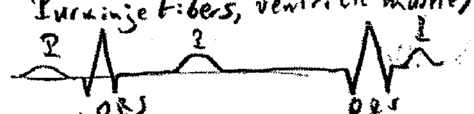
b) $i_{TOT} = C_m \frac{dV_m}{dt} + g_K(V_m - E_K) + g_{Na}(V_m - E_{Na}) + g_{Cl}(V_m - E_{Cl}) + i_p$

c) since $V_m(t) = -60mV + 80mV u(t)$

$C_m \frac{dV_m}{dt} = C_m 80mV \delta(t)$ ⇒ there should be a spike in the current at $t = 0$

Problem 25 (Specialized: Bio Eng-ECE4782) Code Number:__________

a) S/A node, atrial muscles, A/V node, bundle of His, bundle branches, Purkinje Fibers, ventricle muscles

b) 

time between P + QRS is uncorrelated

c) atrial contraction will travel in the opposite direction to the normal so the potential will be determined by a ∇V_m that points in the opposite direction