ECE PHD PRELIMINARY EXAMINATION

SOLUTIONS – SPRING 2008 EXAM

Problem 1 (Core: CompE-ECE2030) Code Number:_________

You are to design a 4-2 priority encoder module. There are four inputs I_0, I_1, I_2, and I_3. The output must encode the subscript of the highest priority asserted (logical 1) input. Inputs with lower subscripts have higher priority. Be sure to correctly handle the case when no inputs are asserted.

a. (3 pts) Derive an implicant table (a truth table that may have don't cares in the input variables) to specify the priority encoder. All outputs necessary to implement the above functionality must be specified.

I_3	I_2	I_1	I_0	Y_1	Y_0	V
0	0	0	0	x	x	0
x	x	x	1	0	0	1
x	x	1	0	0	1	1
x	1	0	0	1	0	1
1	0	0	0	1	1	1

b. (3) Develop minimized logic expressions for the outputs specified in a. You must show your work to receive full credit for the minimization.

Y_1

I_1

	00	11	11	10
00	x			
01	1			
11	1			
10	1			

I_2

I_3

I_0

Y_0

I_1

	00	01	11	10
00	x			1
01				1
11				1
10	1			1

I_2

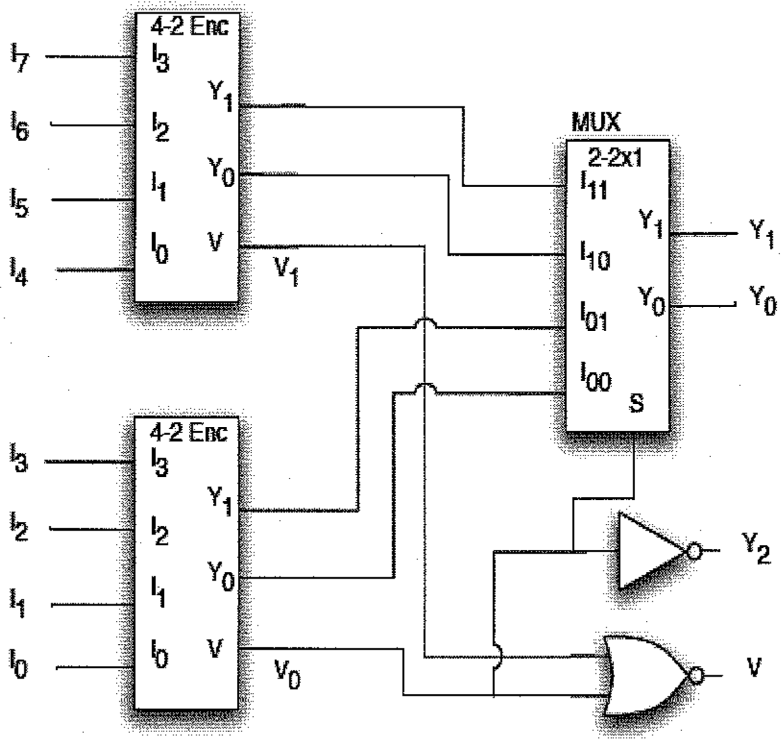
I_3

I_0

$Y_1 = \overline{I_1 I_0}$ $Y_0 = I_1\overline{I_0} + \overline{I_2 I_0}$ $V = I_3 + I_2 + I_1 + I_0$

Problem 1 (Core: CompE-ECE2030) **Code Number:**_________

c. (4) Using a rectangular symbol for the encoder designed above show how to use some number of 4-2 priority encoders to design an 8-3 priority encoder which encodes 8 subscripted inputs ($I_7..I_0$) in 3 bits. As before the output should encode the subscript of the highest priority asserted input, and lower numbered subscripts have higher priority You may use any additional logic gates, multiplexors or decoders that you need.



Problem 2 (Core: CompE-ECE2030) Code Number:__________

a) Using Boolean algebra, simplify this expression such that it has less than or equal to 5 literals. Put in SOP form.

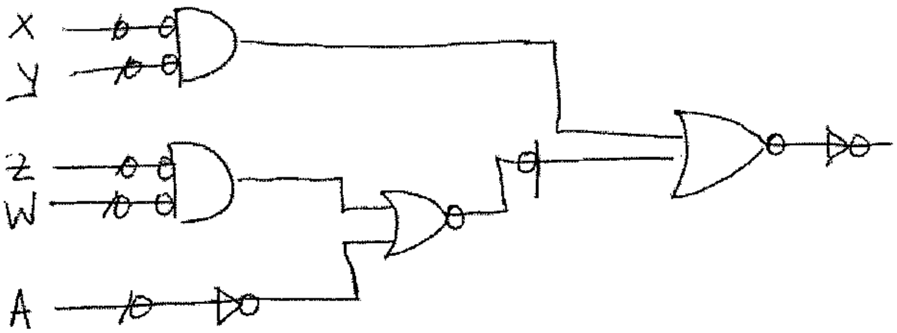
$$OUT = A\overline{B}\;\overline{C} + C + D\overline{A}B + D\overline{A}\;\overline{B}$$

$$(A\overline{B}+C)(\overline{C}+C) + D\overline{A}(B+\overline{B})$$

$$A\overline{B} + C + D\overline{A}$$

b) Show the LOGIC schematic for the following Boolean function. Please show the implementation (preferably with mixed logic notation) using NOR gates and INVERTERS only.

$$F = \overline{X}\;\overline{Y} + \overline{\overline{Z}\;\overline{W} + \overline{A}}$$



c) Represent the following Boolean expression ~~into~~ IN the below k-map. Give the minimized POS and SOP from the k-map.

$$F = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}C + ABCD + \overline{A}\,\overline{B}C\overline{D} + ABD + A\overline{B}C\overline{D} + A\overline{B}\,\overline{C}\,\overline{D}$$

AB \ CD	00	01	11	10
00	1			1
01				
11		1	1	
10	1		1	1

SOP

$$F = \overline{B}\,\overline{D} + ABD + \left\{\begin{matrix} ACD \\ A\overline{B}C \end{matrix}\right.$$

POS

$$F = (A+\overline{B})(\overline{B}+D)(A+\overline{D})(B+C+\overline{D})$$

Problem 3 (Core: CompE-ECE3055) Code Number:__________

The following RISC assembly language program is executed on a 32-bit MIPS processor. Fill in the register values that will be present, after execution of this program. A summary of MIPS instructions is included at the bottom of the page – for anyone unfamiliar with the MIPS instruction set. Prior to execution of the program, memory location 0x01000 contains 0x20313055. ***Note:*** 0x indicates hexadecimal and all answers must be in hexadecimal, default is decimal in the MIPS assembly language source file. A MIPS memory word or register contains 32-bits. Use XXXXXXXX for an undefined value.

```
lw          $3, 0x01000
            srl         $4, $3, 5
            sub         $2, $3, $4
            xor         $3, $4, $3
            lui         $5, 0x8031
            ori         $5, $5, 78

sub         $6, $4, $3
            bne         $3, $6, LABEL1

ori         $6, $0, 20

LABEL1:     sw          $6, 0x01000
```

After execution of the MIPS code sequence above,

R2 = 0x______1f2fa6d3______________(*in hexadecimal*)

R3 = 0x______2130b9d7______________(*in hexadecimal*)

R4 = 0x______01018982______________(*in hexadecimal*)

R5 = 0x______8031004e______________(*in hexadecimal*)

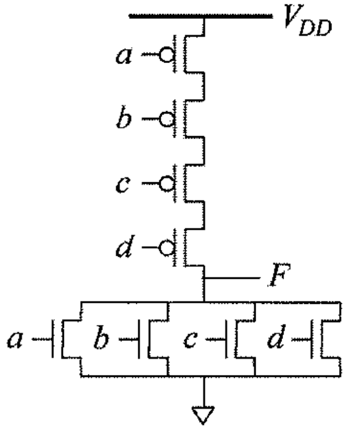
Memory Location 0x01000 contains: 0x_____dfd0cfab_______________(*in hexadecimal*)

Problem 4 (Core: CompE-ECE3060) Code Number:__________

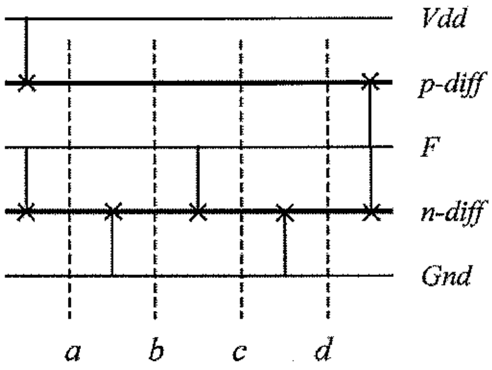
(10 pts) Consider a NOR4 gate $F = \overline{a+b+c+d}$ with the equal rise and fall time. Assume that the ratio between the width of pFET and nFET is 2:1 for an inverter with the equal rise/fall time.

a) (2 pts) Draw the CMOS transistor-level schematic of NOR4.
b) (3 pts) Draw a stick diagram of NOR4.
c) (5 pts) Using the RC delay model, compute the delay of NOR4 driving a load of 100$Cinv$ in term of τ. Assume that the input capacitance of each input is $5Cinv$. $Cinv$ is the input capacitance of the minimum-sized inverter, and $\tau = R \cdot Cinv$. R is the resistance of the minimum-size nFET.

Solutions:



(a)

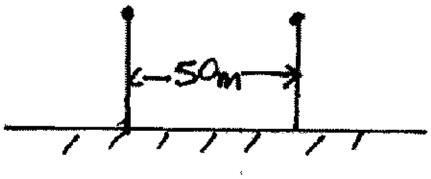


(b)

(c) In a NOR4 with equal rise/fall time, the size ratio between pFET and nFET is 8:1. Since the capacitance of each input is $5Cinv$, each nFET has an input capacitance of $5 \cdot 1/9\, Cinv = 15/9C_{FET}$, where $1Cinv = 3C_{FET}$. Thus, the resistance of nFET is $9/15R$. Lastly, the RC delay is $9/15R \cdot 100Cinv = 60\tau$.

Problem 5 (Core: E&M-ECE3025) **Code Number:__________**

A standard AM broadcast band transmitting station consists of two vertical monopoles above the earth. The two antennas are separated by 50 m and the transmitting frequency is 1500 KHz. The antennas are fed with signals of equal amplitude and a phase difference of 135°. Sketch the electric field pattern at the surface of the earth. Show the location of all maxima and minima and their relative values.



$$\lambda_0 = \frac{c}{f_0} = 200\,m$$

$$d = 50\,m = \frac{\lambda_0}{4}, \quad \alpha = 135^\circ$$

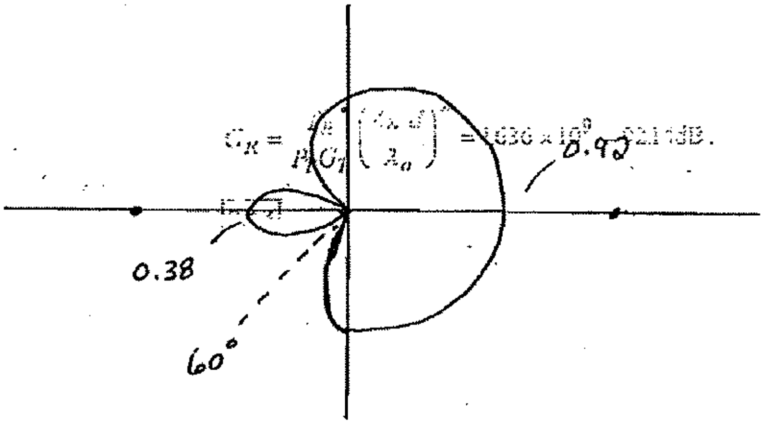
$$E \propto \cos\left[\frac{\pi d}{\lambda_0}\cos\phi + \frac{\alpha}{2}\right] = \cos\left[\frac{\pi}{4}\cos\phi + \frac{3\pi}{8}\right]$$

Nulls at $\frac{\pi}{4}\cos\phi + \frac{3\pi}{8} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$ or

$$\cos\phi = 0.5, -3.5 \text{ or } \phi = 60^\circ$$

Maxima & minima at $\sin\left(\frac{\pi}{4}\cos\phi + \frac{3\pi}{8}\right) = 0$ or

$$\frac{\pi}{4}\cos\phi + \frac{3\pi}{8} = 0, \pm\pi \rightarrow \phi = 0^\circ \;\&\; \phi = 180^\circ$$



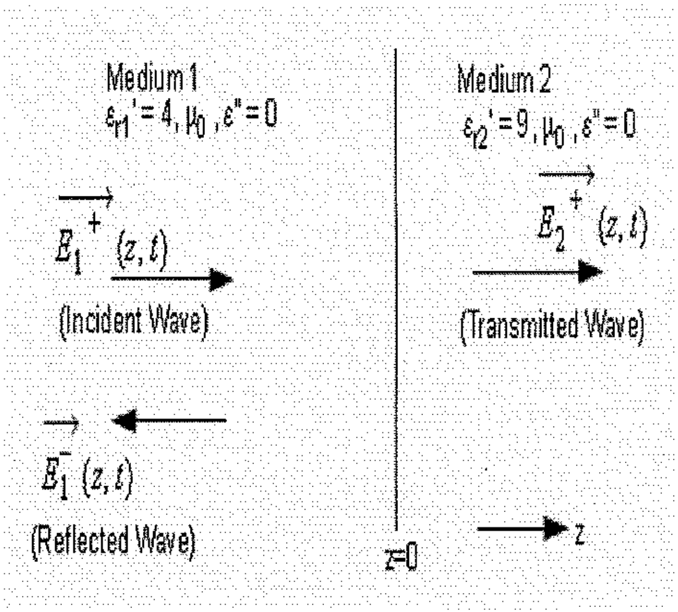
Problem 6 (Core: E&M-ECE3065) Code Number:__________

Problem 2

Consider two lossless, non-magnetic, dielectric media as shown in the figure below. The interface between the two media is on the x-y plane at z=0. A uniform plane wave traveling in medium 1 is incident normally at the interface. The electric field of the incident wave is given as:

$$\vec{E}^{+}(z,t) = 10Cos(\omega t - \pi z)a_x \quad (\frac{V}{m})$$

This wave generates a reflected wave in medium 1 and a transmitted wave in medium 2, as shown in the figure below. The relative permittivity of medium 1 and 2 are ε_{r1}'=4 and ε_{r2}'=9, respectively.



a) Calculate the frequency of the wave.

b) Calculate the minimum distance from the interface at which the total magnitude of electric field in Medium 1 is a maximum.

c) Calculate the standing wave ratio of the wave in medium 1.

d) Calculate the electric field of the transmitted wave in medium 2 at a distance of z=1m from the interface and at time t=13.33ns.

e) Calculate the average power density of the transmitted wave in medium 2.

Problem 1 : Answer

a) Medium 1

$$\beta = \pi \Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\pi} = 2$$

$$\therefore f = \frac{3\times10^8}{\sqrt{4}\times 2} = \frac{3\times10^8}{4} = \boxed{75\,MHz}$$

b) $\vec{E}^{+}_{xs1} = 10e^{-j\pi z} a_x \; (V/m)$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{120\pi}{\sqrt{9}}-\frac{120\pi}{\sqrt{4}}}{\frac{120\pi}{\sqrt{9}}+\frac{120\pi}{\sqrt{4}}} = \frac{40\pi-60\pi}{40\pi+60\pi} = -\frac{1}{5}$$

$$\vec{E}^{-}_{xs1} = 10\times\left(-\frac{1}{5}\right)e^{+j\pi z}$$

$$\vec{E}_{xs1} = 10\left\{e^{-j\pi z} + \frac{1}{5}e^{+j(\pi z+\pi)}\right\}$$

$$\left|\vec{E}_{xs1,max}\right| = 10\left\{1+\frac{1}{5}\right\} \quad \text{when} \quad -\pi z = \pi z+\pi$$

$$\therefore -2\pi z = \pi \Rightarrow \boxed{z = -\frac{1}{2}\,m}$$

c) $$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.2}{1-0.2} = \boxed{1.5}$$

d) Medium 2

$$\beta_2 = \frac{2\pi}{\lambda} \; ; \; \lambda = \frac{3\times10^8}{\sqrt{9}\times75\times10^6} = 1.33\,(m)$$

$\beta_2 = \frac{2\pi}{1.33} = 1.5\pi$

$\tau = 1 + \Gamma = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore \vec{E}^{+}_{xs2} = \frac{4}{5} e^{-j1.5\pi z} a_x \quad (\frac{V}{m})$

$\therefore \vec{E}_2(z,t) = \frac{4}{5} \cos(\omega t - 1.5\pi z) a_x \quad (\frac{V}{m})$

At $z = 1m$; $t = 13.33 ns$

$\vec{E}_2(z,t) = \frac{4}{5} \cos(2\pi \times 75 \times 10^6 \times 13.33 \times 10^{-9} -$
$1.5\pi \times 1) a_x \quad (\frac{V}{m})$

$= \frac{4}{5} \cos(0.5\pi) a_x = \boxed{0}$

e) $P_{avg,2} = \frac{1}{2} Re\{\vec{E}_2 \times \vec{H}_2^*\}$

$\vec{H}_2 = \frac{1}{\eta_2} a_z \times \vec{E}^{+}_{xs2} = \frac{1}{\frac{120\pi}{\sqrt{9}}} a_y \frac{4}{5} e^{-j1.5\pi z}$

$\therefore \vec{H}_{ys2} = \frac{3}{120\pi} \times \frac{4}{5} e^{-j1.5\pi z} a_y = \frac{0.02}{\pi} e^{-j1.5\pi z} a_y$

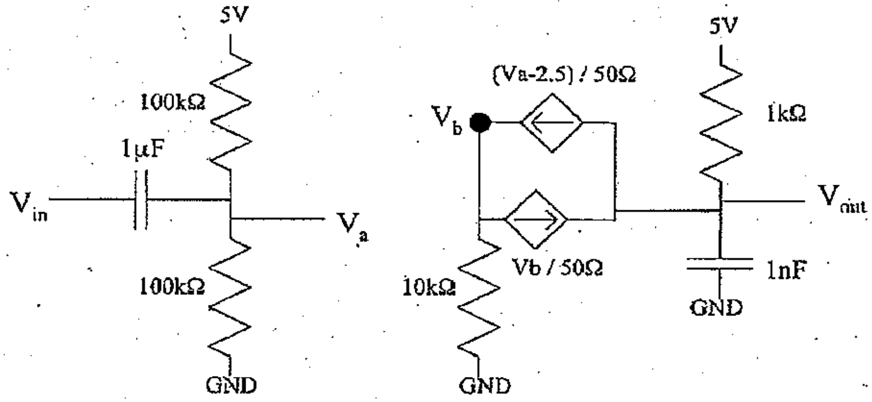
$\therefore P_{avg,2} = \frac{1}{2} Re\left\{ \frac{4}{5} e^{-j1.5\pi z} a_x \times \frac{0.02}{\pi} e^{+j1.5\pi z} a_y \right\}$

$\therefore P_{avg,2} = \frac{1}{2} \times 0.005 = \boxed{2.5 mW}$

Problem 7 (Core: EDA-ECE2040) Code Number:__________

2040 Prelim Question:

Answers must be written on the line to receive any credit.



The following questions relate to the DC operation of this circuit. Assume we apply a dc input (Vin) voltage of 3.7V.

DC voltage at Va = <u>2.5V (Cap has no effect)</u> DC voltage at Vb = <u>0V (KCL at Vb)</u>

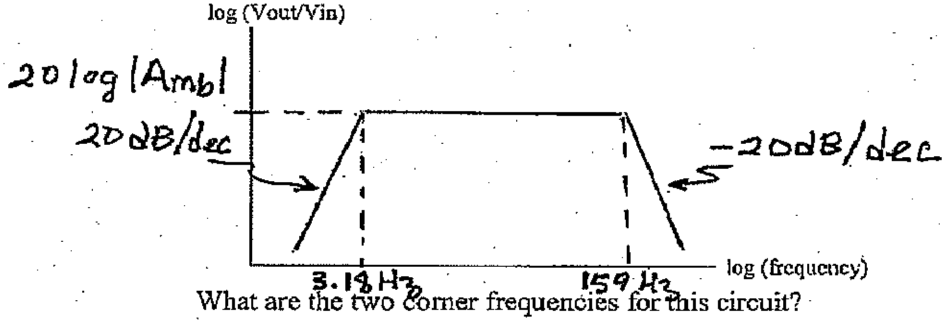
DC voltage at Vout = <u>5V (no I drop through 1kOhm resistor)</u>

What condition does one use to calculate the dc parameters? <u>Capacitors conduct no I</u>

The following questions relate to the AC operation of this circuit. Assume we apply in input signal, Vin = (10mV) sin ωt, where ω is the input frequency of the signal.

What is the gain between Va and Vb? = <u>1/(1 + (50Ω /10kΩ)) = 0.995 (KCL at Vb)</u>

Plot the frequency response of the circuit, namely log| Vout(jω) / Vin(jω) | versus log(frequency). Identify appropriate breakpoints and other key aspects in the graph.



What are the two corner frequencies for this circuit?

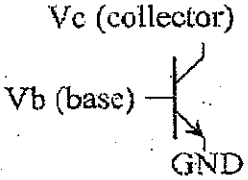
<u>1/(2π 50kΩ 1μF) = 3.18 Hz</u> and <u>1/(2π 1kΩ 1vF) = 159 kHz</u>

□

What is the midband gain for this circuit? <u>-(1kΩ / 10kΩ) = -0.1</u>

Problem 8 (Core: EDA-ECE3050) Code Number:__________

3050 Prelim Question:

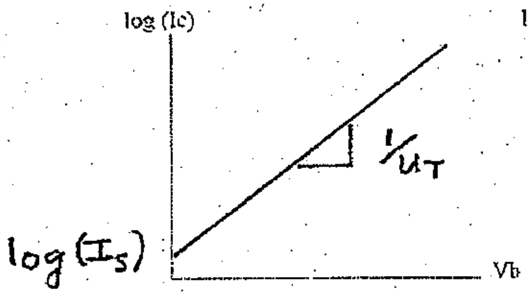


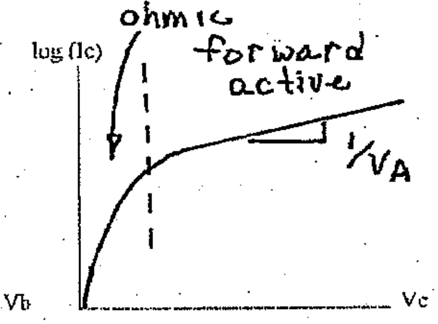
Assume the BJT devices have a "Beta" that is infinite, that Is = 1pA, and that the collector and emitter dopings are identical (you might identify this device similar to a subthreshold MOSFET with "kappa" =1).

Write the (large-signal) expression for the collector current (Ic) versus Vb and Vc

Ic = _Is exp(Vb / UT) (1 – exp(-Vc/UT)) (ohmic region)
Ic = _Is exp(Vb / UT) (1 + (Vc / VA)) (forward-active region)
UT = kT/q ~ 25mV at room temperature, VA = Early voltage

Plot log Ic versus Vb, and log Ic versus Vc. Identify key breakpoints on graph.

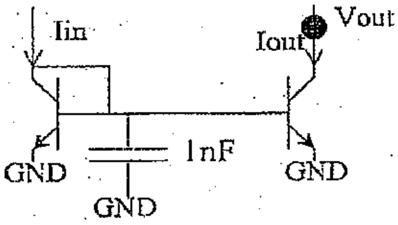




What is the slope of the curve (log I versus Vb) at 1µA? 1 / UT = 40 V^{-1}

What is the slope of the curve (I versus Vb) at 1µA? 1µA / UT = 40 µA/V

What voltage for Vc defines the boundary between the ohmic and forward active region? 4 UT = 100mV



Assume the transistors are identical.

What is the gain (Iout/Iin) for this circuit when Iin = 1µA? 1

How low can Vout decrease and the corresponding transistor act like a voltage-controlled current source? 4UT = 100mV

For a small signal input riding on a dc current of 1µA, what is the low-pass corner frequency of this circuit? 1µA / (2 π 1nF UT) = 6.37kHz

Problem 9 (Core: Power-ECE3070) **Code Number:**__________

An electromechanical actuator with a single coil and an air gap has a λ-i relationship given by,

$$\lambda = \frac{1.2i^{1/2}}{g}$$

where λ is the flux linkages of the coil, i is the current in the coil, and g is the air gap length. Assume that there is a dc current in the coil of 2 A.

If the air gap length is 10 cm …

(a) Determine the energy stored in the magnetic field.
(b) Determine the force acting on the air gap.

Solution

(a) The energy stored in the magnetic field is given by,

$$W_m = \frac{1}{2}\lambda i = \frac{1}{2} i \frac{1.2i^{1/2}}{g} = \frac{1.2}{2}\frac{i^{3/2}}{g} = \frac{1.2}{2}\frac{2^{3/2}}{.1} = 17\,\text{J}$$

(b) The force on the air gap is given by,

$$F = \frac{dW_m}{dg} = \frac{1}{2}\frac{d(\lambda i)}{dg} = \frac{1}{2} i \frac{d\lambda}{dg}$$

$$= \frac{1}{2} i \frac{d}{dg}\left(\frac{1.2i^{1/2}}{g}\right) = \frac{1}{2} i \left(-\frac{1.2i^{1/2}}{g^2}\right) = \frac{1}{2} 2 \left(-\frac{1.2(2)^{1/2}}{.1^2}\right) = -170\,\text{N}$$

Problem 10 (Core: Power-ECE3070) Code Number:__________

The per phase synchronous reactance of a three-phase, Y-connected, 60 Hz, 4-pole, synchronous generator is 10 ohms. When a certain load is connected to the generator terminals, it supplies 2.5 MVA (total three phase) at 60 Hz, at a leading power factor of 0.8 and a terminal voltage of 6.6 kV (L-L), to the load. Neglect armature resistance, magnetic saturation, and all mechanical losses. For the above load condition:

a) Find the speed of the generator.

b) Calculate the mechanical torque supplied by the turbine which drives this generator.

c) Draw the phasor diagram showing $\widetilde{V}_t$, $\widetilde{E}_{af}$, $\widetilde{I}_a$ and $jX_s\widetilde{I}_a$. Recall that $\widetilde{V}_t = \widetilde{E}_{af} - jX_s\widetilde{I}_a$. Use V_t as the reference phasor. Calculate the magnitude (expressed as a L-N value) of the excitation voltage (or induced voltage) $\mathbf{E_{af}}$.

d) The load is now removed from the generator terminals and the field current and speed of the generator remain unchanged. Find the new value of the terminal voltage expressed as a Line-to-Neutral (L-N) value.

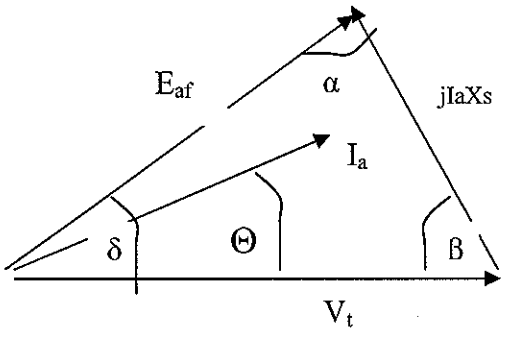
<u>SOLUTION</u>

(a) The output frequency of a synchronous generator is proportional to the speed. For a 4 pole 60 Hz machine, the speed is 1800 rpm.

(b) The mechanical speed is 188.5 rad/sec. With no losses the input shaft power is (2.5 x 0.8)MW

Thus torque is (2500 x 0.80)/188.5 = 10.6 k Nm

(c) Phasor diagram:

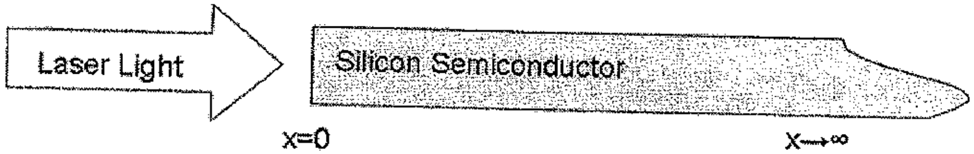


Per phase terminal voltage $V_t = (6.6 \text{ x } 1000)/1.732 = 3811$ volts
Armature current per phase $I_a = (2500 \text{ x } 1000)/ (1.736 \text{ x } 6600) = 218.7$ A

$\widetilde{E}_{af} = 3811 + j218.7(0.8 + j0.6)(10) = 3050$ at an angle of 35 degrees. So the magnitude of this voltage is 3050 volts.

(d) The new value of the terminal voltage will be 3050 volts per phase.

Problem 11 (Core: Microsystems-ECE3040) Code Number:_________



A semi-infinite (with limits at x=0 and x=infinity) slab of silicon semiconductor (energy bandgap = 1.1 eV, intrinsic concentration = 1e10 cm-3, and donor doped to 6e17 cm-3, minority carrier lifetime = 1e-5 seconds, and minority carrier mobility = 500 cm2/Vsec) is illuminated by a high photon energy laser (each photon having 4 eV of energy) such that most all of the light is absorbed in a thin layer near the surface approximated as a two dimensional sheet. If the continuous, time independent light results in a steady state hole concentration at the illuminated surface (i.e. at x=0) of 1e12 cm-3, solve the minority carrier diffusion equation (showing your work) to result in an equation describing the hole concentration at all positions x.

Minority Carrier diffusion Equation:

$$\frac{d\Delta p}{dx} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau} + G_L$$

For $x > 0$ + CW light

$$0 = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau}$$

General Solution: $\Delta p(x) = A e^{-x/L_p} + B e^{+x/L_p}$

~~BC~~ Boundary Conditions:

$\Delta p(x=0) = 10^{12}\ cm^{-3}$

$\Delta p(x=\infty) = 0 \implies B = 0$

$$\Delta p(x) = (10^{12} = A) e^{-x/L_p}\ cm^{-3}$$

$L_p: = \sqrt{D_p \tau_p}$ $\qquad D_p = \frac{kT}{q}\mu_n \equiv$ Einstein Relationship

$= \sqrt{12.95(1e\text{-}5)}$ $\qquad = 0.0259(500)$

$= 0.0113\ cm$ $\qquad = 12.95$

$$\therefore \Delta p(x) = \left[(1e12)\, e^{-x/0.0113cm}\right] cm^{-3}$$

Problem 12 (Core: Microsystems-ECE3080) Code Number:_________

SOLUTION:

(a) *Yes, equilibrium conditions prevail inside the semiconductor, because the Fermi level* ***inside the semiconductor*** *is position independent.*

(b) *From the graph, we obtain* $\phi_F = \frac{1}{q}\left[E_{i,bulk} - E_F\right] = \frac{1}{q}\left[0.30\ eV\right] = 0.3\ V$

(c) *Again, from the graph, we obtain* $\phi_S = \frac{1}{q}\left[E_{i,bulk} - E_{i,surface}\right] = \frac{1}{q}\left[0.30\ eV\right] = 0.3\ V$

(d) *Again, from the graph, we obtain with*

$$E_{F,metal} - E_{F,semi} = -qV_G$$

$$V_G = \frac{1}{q}\left[E_{F,semi} - E_{F,metal}\right] = 0.6V$$

(e) *Based on the delta-depletion approximation, we have*

$$V_G = \phi_S + \frac{K_S x_o}{K_o}\sqrt{\frac{2qN_A}{K_S \varepsilon_0}\phi_S}$$

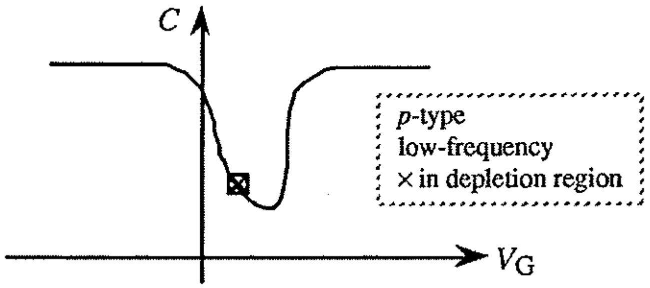
Using the known Fermi potential, we can calculate the doping concentration NA

$$N_A = n_i\, e^{(E_{i,bilk}-E_F)/kT} = n_i\, e^{\phi_F/qkT} = 1.07\,10^{15}\ cm^{-3}$$

yielding the oxide thickness

$$x_o = \frac{V_G - \phi_S}{\frac{K_S}{K_o}\sqrt{\frac{2qN_A}{K_S\varepsilon_0}\phi_S}} = 0.1\ \mu m$$

(f)



Problem 13 (Core: DSP-ECE2025) Code Number:__________

POSSIBLE QUESTIONS and CLARIFICATIONS:

1. The *areas of the impulses* are written with a factor of 2π to call attention to the fact that the FT of a complex exponential, $e^{j\omega_0 t}$, is 2π times an impulse located at $\omega = \omega_0$, i.e., $2\pi\delta(\omega - \omega_0)$.
2. The symbol ω is for radian frequency in rad/s, but note that the sampling frequency is given in hertz.
3. The system choices can be used more than once.
4. The answer for each part is unique.
5. There are no spectrum components outside of the region $[-5\pi, 5\pi]$ rad/s.
6. The numbers are correct—some are quite close to one another to provide confusion.

PROBLEM SOLUTION:

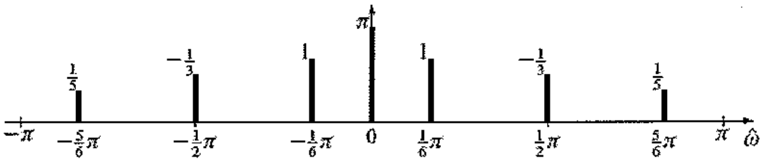
For the first two parts, we map the continuous-time frequency ω to the discrete-time frequency $\hat{\omega}$ via

$$\hat{\omega} = \frac{\omega}{f_s} + 2\pi\ell \qquad \text{where } \ell = 0, \pm 1, \pm 2 \ldots.$$

(a) For $f_s = 6$ Hz there is no aliasing because f_s is more than the Nyquist rate, $\frac{\omega_{\max}}{2\pi} = \frac{5\pi}{2\pi} = 2.5$ Hz. Thus the discrete-time frequencies are

$$\hat{\omega} = \frac{\omega}{f_s} = \frac{\omega}{6} \qquad \Rightarrow \quad \{0, \pm\pi/6, \pm\pi/2, \pm 5\pi/6\}$$

The complex amplitudes for the spectrum of $x[n]$ are the impulse areas divided by 2π.

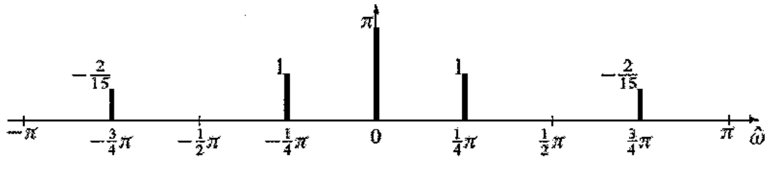


(b) For $f_s = 4$ Hz there *will be aliasing* because f_s is less than the Nyquist rate of 2.5 Hz. The discrete-time frequencies are

$$\hat{\omega} = \frac{\omega}{f_s} = \frac{\omega}{4} \qquad \Rightarrow \quad \{0, \pm\pi/4, \pm 3\pi/4, \pm 5\pi/4\}$$

but when aliasing is taken into account, the frequency $\hat{\omega} = 5\pi/4$ aliases to $\hat{\omega} = -3\pi/4$. Thus, the spectrum of $x[n]$ contains the following frequencies: $\{0, \pm\pi/4, \pm 3\pi/4\}$. The complex amplitudes for the spectrum of $x[n]$ are the impulse areas divided by 2π, but the complex amplitude for $\hat{\omega} = 3\pi/4$ comes from $\omega = -3\pi$ and $\omega = 5\pi$. Thus, we must add the complex amplitudes for $\omega = -3\pi$ and $\omega = 5\pi$.

$$\frac{\left(-\frac{2\pi}{3} + \frac{2\pi}{5}\right)}{2\pi} = -\frac{2}{15}$$



PROBLEM SOLUTION:

The approach to solving the matching problem will be to treat the two sampling frequencies as separate cases, and generate the outputs for all seven systems. Then, from those fourteen results for $y(t)$, the matched answers can be picked.

For $f_s = 6\,\text{Hz}$ we can use the spectrum of $x[n]$ to write the output of the C-to-D converter as a sum of sinusoids

$$x[n] = \pi + 2\cos(\pi n/6) - \frac{2}{3}\cos(3\pi n/6) + \frac{2}{5}\cos(5\pi n/6)$$

and we note that the spectrum of $x[n]$ contains the following frequencies: $\{0, \pm\pi/6, \pm\pi/2, 5\pi/6\}$. Thus we need to evaluate the frequency responses of the digital filters at these frequencies, and we summarize the results in a table:

	$\hat{\omega} = 0$	$\hat{\omega} = \pi/6$	$\hat{\omega} = \pi/2$	$\hat{\omega} = 5\pi/6$	$y[n]$
1.	0	1	1	1	$2\cos(\pi n/6) - \frac{2}{3}\cos(3\pi n/6) + \frac{2}{5}\cos(5\pi n/6)$
2.	1	$e^{-j2\pi/6}$	$e^{-j\pi}$	$e^{-j10\pi/6}$	Delay by two, so $y[n] = x[n-2]$
3.	1	$e^{-j3\pi/6}$	$e^{-j3\pi/2}$	$e^{-j15\pi/6}$	Delay by three, so $y[n] = x[n-3]$
4.	0	0	1/5	0	$-\frac{2}{15}\cos(3\pi n/6)$
5.	1	1	0	0	$\pi + 2\cos(\pi n/6)$
6.	1	$e^{-j\pi/6}$	$e^{-j\pi/2}$	$e^{-j5\pi/6}$	Delay by one for the four freqs., so $y[n] = x[n-1]$
7.	0	0	0	1	$\frac{2}{5}\cos(5\pi n/6)$
$x[n]$	π	2	$-2/3$	$2/5$	← sinusoidal amplitudes

The mapping from n back to t is $n = (f_s)t = 6t$; and the mapping from $\hat{\omega}$ back to ω is $\omega = (f_s)\hat{\omega} = 6\hat{\omega}$. In addition, a delay of one sample in the discrete-time system is equivalent to a delay of $1/f_s$ for $y(t)$, i.e., $y(t) = x(t - \frac{1}{6})$. Now we can make a table with all the outputs:

1.	$2\cos(\pi t) - \frac{2}{3}\cos(3\pi t) + \frac{2}{5}\cos(5\pi t) = x(t) - \pi$
2.	Delay by two, so $y(t) = x(t - 2/6) = y(t - \frac{1}{3})$
3.	Delay by three, so $y(t) = x(t - 3/6) = y(t - \frac{1}{2})$
4.	$-\frac{2}{15}\cos(3\pi t)$
5.	$\pi + 2\cos(\pi t)$
6.	Delay by one, so $y(t) = x(t - \frac{1}{6})$
7.	$\frac{2}{5}\cos(5\pi t)$

Thus, part (c) is System #1 and part (e) is System #2.

PROBLEM SOLUTION:

For $f_s = 4\,\text{Hz}$ we again use the spectrum of $x[n]$ to write the output of the C-to-D converter as a sum of sinusoids

$$x[n] = \pi + 2\cos(\pi n/4) - \frac{4}{15}\cos(3\pi n/4)$$

Recall that the spectrum of $x[n]$ contains only the following frequencies: $\{0,\ \pm\pi/4,\ \pm 3\pi/4\}$. Thus we need to evaluate the frequency responses of the digital filters at these frequencies, and we summarize the results in a table:

	$\hat{\omega} = 0$	$\hat{\omega} = \pi/4$	$\hat{\omega} = 3\pi/4$	$y[n]$
1.	0	1	1	$2\cos(\pi n/4) - \frac{4}{15}\cos(3\pi n/4)$
2.	1	$e^{-j2\pi/4}$	$e^{-j6\pi/4}$	Delay by two, so $y[n] = x[n-2]$
3.	1	$e^{-j3\pi/4}$	$e^{-j9\pi/4}$	Delay by three, so $y[n] = x[n-3]$
4.	0	0	0	0
5.	1	0	0	π
6.	1	$e^{-j\pi/4}$	$e^{-j3\pi/4}$	Delay by one for the three freqs., so $y[n] = x[n-1]$
7.	0	0	1	$-\frac{4}{15}\cos(3\pi n/4)$
$x[n]$	π	2	$-4/15$	← sinusoidal amplitudes

The mapping from n back to t is $n = (f_s)t = 4t$; and the mapping from $\hat{\omega}$ back to ω is $\omega = (f_s)\hat{\omega} = 4\hat{\omega}$. In addition, a delay of one sample in the discrete-time system is equivalent to a delay of $1/f_s$ for $y(t)$, i.e., $y(t) = x(t - \frac{1}{4})$. Now we can make a table with all the outputs:

1.	$2\cos(\pi t) - \frac{4}{15}\cos(3\pi t)$
2.	Delay by two, so $y(t) = x(t - 2/4) = x(t - \frac{1}{2})$
3.	Delay by three, so $y(t) = x(t - \frac{3}{4})$
4.	0
5.	π
6.	Delay by one, so $y(t) = x(t - \frac{1}{4})$
7.	$-\frac{4}{15}\cos(3\pi t)$

Thus, part (d) is System #4 and part (f) is System #7.

Problem 14 (Core: DSP-ECE3075) Code Number:__________

(a) Find the marginal densities of X and Y.

Marginals of Jointly Gaussian Random Variables (JGRV's) are also Gaussian $\longrightarrow$ $\bar{X} = 0$ $\sigma_X^2 : 9 \cdot 0^2 = 9$

$\bar{Y} = 2$ $\sigma_y^2 = 8 \cdot 2^2 = 4$

$X \sim N(0, 9)$; $Y \sim N(2, 4)$

(b) Find the correlation coefficient, ρ, between X and Y.

$$\rho : \frac{E(XY) - E(X)E(Y)}{\sigma_x \sigma_y} = \frac{-1}{(3)(2)} = -\frac{1}{6}$$

(c) Find the probability that $-1 \leq X \leq 1$. Express this in terms of $\Phi(\alpha)$, the cumulative distribution function of a zero mean, unit variance Gaussian random variable. A table is attached.

$Prob(-1 < X < 1)$

$= \Phi(\frac{1}{3}) - \Phi(-\frac{1}{3})$

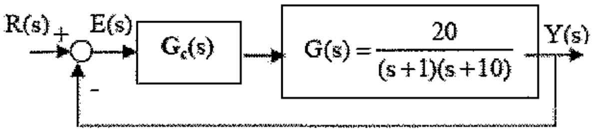
$= 2\Phi(\frac{1}{3}) - 1$

But the table only finds the area to the right of 0. $\Phi(\frac{1}{3}) = .5 + .13 = .63$ $\longrightarrow$ $Prob(\cdot) = .26$

Problem 15 (Core: S&C-ECE3085) Code Number:__________

Systems and Controls Preliminary **Solution**
Spring 2008

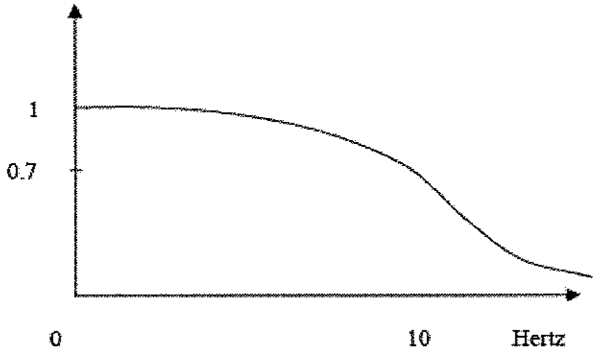
Consider a unity feedback system in the figure.



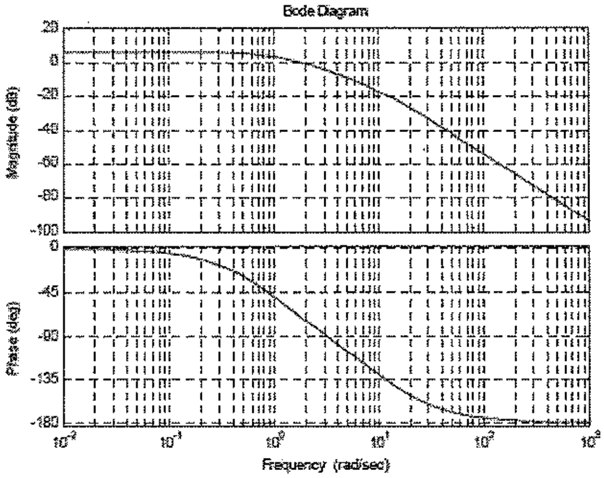
where $G_c(s)$ represents the compensator. The specifications on the closed loop system are to have zero steady-state error for a set point command and no more than 30% steady-state error for sinusoidal inputs up to 10 Hz, where a set point command can be considered as a step input.

a) Sketch the frequency response of the closed loop system that satisfies these conditions. Use linear scales.
b) Sketch the Bode plot of the open loop system without the compensator.
c) Select a simple compensator that will satisfy these conditions.

a) The desired closed loop frequency response is a lowpass filter with bandwidth 10Hz.



b)



c) A PI controller is needed. A simple one would cancel the pole closest to the origin, so $G_c(s) = \frac{K(s+1)}{s}$.

With this control, the open loop system is $G_c(s)G(s) = \frac{K20}{s(s+10)}$ and the closed loop system is

$$G_c(s) = \frac{K20}{s^2 + 10s + K20}.$$

Let $\omega_n = 10(2\pi)\text{rad/sec}$.

$$\omega_n = \sqrt{K20} = 20\pi \Rightarrow K = 20\pi^2 \approx 197$$

Solution

a

We have

$$\begin{aligned} Y(s) &= G_2(s)(V(s) + G_1(s)U(s)) \\ &= G_2(s)(V(s) - G_1(s)C(s)Y(s)) \\ (1 + G_1(s)C(s))Y(s) &= G_2(s)V(s) \\ Y(s) &= \frac{G_2(s)}{1 + G_1(s)C(s)}V(s), \end{aligned}$$

and hence

$$G(s) = \frac{G_2(s)}{1 + G_1(s)C(s)}.$$

b

We note that C_1 and C_2 are P-regulators, while C_3 and C_4 are PI-regulators.

In the PI-case, we get

$$Y(s) = \frac{4(s+2)}{s^2 + 3s + 2 + 8K_P}V(s),$$

with roots to the characteristic equation given by

$$s = -1.5 \pm \sqrt{0.25 - 8K_P}.$$

And hence the system is asymptotically stable for all $K_P > 0$.

The final value theorem gives that

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} \frac{1}{s} s \frac{4(s+2)}{s^2 + 3s + 2 + 8K_P} = \frac{4}{1 + 4K_P}.$$

In other words, the disturbance is not perfectly suppressed. But, a higher gain gives a smaller steady state value.

In the PI-case, we get

$$Y(s) = \frac{4s(s+2)}{s^3 + 3s^2 + (2 + 8K_P)s + 8K_I}V(s).$$

Routh's criterium gives

$$\begin{array}{cc} 1 & 2 + 8K_P \\ 3 & 8K_I \\ (24K_P + 6 - 8K_I)/3 & 0 \\ 8K_I & 0 \end{array}$$

Hence we see that the stability condition is that $K_I > 0$ and $24K_P + 6 - 8K_I > 0$, which is satisfied by C_3 but not by C_4.

If the system is stable, the final value theorem gives

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} \frac{1}{s} s \frac{4s(s+2)}{s^3 + 3s^2 + (2 + 8K_P)s + 8K_I} = 0.$$

We can thus draw the conclusion that

C_1 → Step Response 3

C_2 → Step Response 1

C_3 → Step Response 2

C_4 → Step Response 4

Problem 17 (Specialized: Comp Science-CS3210) Code Number:______

Consider a virtual memory system for a 64-bit processor with a 40 bit virtual address space and a 1 GByte real address space with 16 Kbyte pages and a 16 entry fully associative TLB.

a) Provide a definition of the working set of a program.

The working set of a process in an interval is the set of pages referenced by the process in that interval.

b) With a 64-bit page table entry, what is the size in bytes of the flat page table?

512Mbytes $= 2^{26}$ entries * 2^3 bytes/entry

c) What is the function of the TLB and what is the difference between a TLB miss and a page fault.

The TLB is a cache of page table entries for translating virtual addresses to physical addresses. A TLB miss simply means that a page table entry is not in the TLB. A page fault on the other hand occurs when the requested page is not in memory.

d) With the preceding memory system we have four processes operating in a time shared manner and requesting physical memory in units of 64 pages. The current allocation in Mbytes and the maximum amount that can be requested by each process is shown in the table below. Deadlock occurs when processes block indefinitely waiting for sufficient memory to become available to complete execution.

Process	**Allocated**	**Maximum**
A	192	512
B	320	384
C	128	192
D	128	384

Define an algorithm for memory allocation that will avoid deadlock while having processes continue executing in a time-shared manner, if the sequence of memory requests permits. Using your algorithm, if C makes a request for 64 Mbytes can it be allocated?

Solution:
This is one of several possible solutions.

The criterion for deadlock freedom is that at least one process can request and be allocated enough memory for completion. When this process completes and de-allocates memory, another process will be able to complete and so on where all processes can complete execution. The following algorithm checks for deadlock freedom where Max_i is the maximum request process *i* can make in the future (Allocated – Maximum)

Deadlock-free ().

1. Set `free` to be the total available space in Mbytes (assuming this request is granted)

2. If there exists a process for which Max_i <= `free`, `free` = `free` + Allocated_i and the process is marked as safe. Repeat this step until no more safe processes are found.

3. If all processes are safe then, commit the allocation and return true, else return false.

Based on this allocation, C's request can be granted.

Problem 18 (Specialized: Software Sys- ECE3035) Code Number: _____

Solution to Prelim Question: Computer Science Specialized Area (Blough)

Using C-like pseudo-code, write a routine insert(**x**, head) that inserts an integer into a singly-linked list in sorted order (from least to greatest), where head is a pointer to the first item on the list (or NULL if the list is empty). Make sure to include code to allocate memory for a new item when inserting. Assume that the list contains no duplicate values, i.e. if **x** is already in the list, then the insert routine should not insert anything.

```
struct node {
        int num;
        struct node * next;
}

struct node * insert (int x, struct node * head)
{
        struct node * temp, last;

        temp=last=head;
        if (head == NULL) head = insertnode(x, head);
        else {
                while (temp != NULL) {
                        if ((temp->num > x) AND (last->num < x)) last = insertnode(x, last);
                        last = temp;
                        temp = temp->next;
                }
                if (last->num < x) last = insertnode(x, last);
        }
        return head;
}

struct node * insertnode (int x, struct node * here)
{
        struct node * temp;

        temp = (struct node *) malloc(sizeof(struct node));
        temp->num = x;
        temp->next = NULL;
        if (here != NULL) {
                temp->next = here->next;
                here->next = temp;
        }
        else here = temp;
        return here;
}
```

Problem 19 (Specialized: Telecom-ECE3076) Code Number:__________

Solution:

What four items of information are needed by a host before it can operate normally on the network:

1. __**Assigned IP address**___
2. __**Network mask**___
3. ___**Gateway router IP address**____
4. ___**DNS server IP** _____

If these are not configured manually, what protocol can be used to get them over the network?

5. _**DHCP**_______________________________

Before sending an IP datagram, what protocol is used to find the right Ethernet address?

6. ___**ARP**_______________________________

What Ethernet address is used for a host not on the local network?

7. ___ **Gateway router address** ___

If a host knows its own IP address, what does it need to calculate its Network IP and Broadcast IP?

8. _____**Network mask**__

How does (9) CIDR and (10) NAT lead to much more efficient use of the available IP address space?

9. __________ **CIDR allows the assigned network IP-address range to be tailored to fit the need.**__

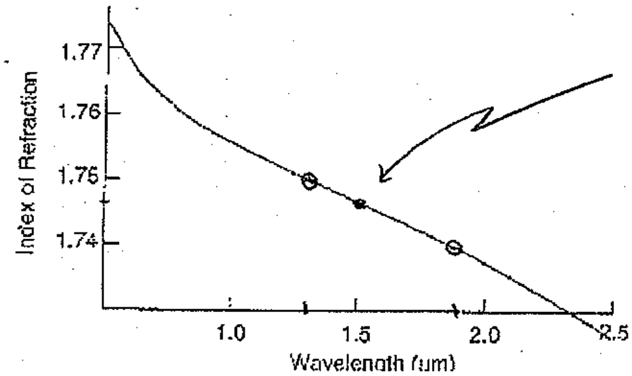
10. ________ **NAT allows use of a single external address for private networks.**_____ ______

Problem 20 (Specialized: Optics – ECE 4500) **Code Number:** ___________

Waves

For a wave with free space wavelength 1500nm and propagating in a material with index of refraction as shown

a) Calculate the wave parameters ν, λ, and k of a wave propagating in the medium
b) Determine the phase and group velocities at 1550nm
c) Sketch the group velocity vs wavelength over the range of data shown



$n \approx 1.747$

$\frac{dn}{d\lambda} \cong \frac{1.74-1.75}{1.8_{\mu}-1.3_{\mu m}} = \frac{-.01}{500nm} = \underline{-2\times10^{-5}\frac{1}{nm}}$

a) $\lambda_o = 1500nm$ $n @ \lambda = 1500nm \cong 1.747$

$\nu\lambda_o = c \Rightarrow \nu = c/\lambda = \underline{\underline{200}}\, THz$; $\lambda = \lambda_o/n = \frac{1500}{1.747} = \underline{\underline{858nm}}$

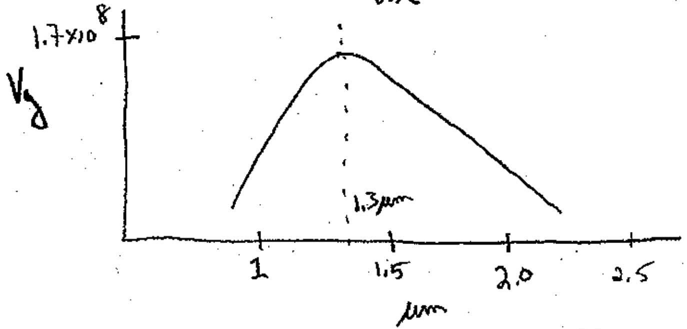
$K = \frac{2\pi}{\lambda_o/n} = \frac{2\pi}{\lambda} = \frac{2\pi}{858\times10^{-9}} = \underline{\underline{7.32\times10^{6}}}\ 1/m$

b) $V_p = c/n = \frac{3\times10^{8}}{1.747} = \underline{\underline{1.72\times10^{8}}}\ m/s$

$V_g = \frac{c}{n - \lambda\, dn/d\lambda} = \frac{c}{N_g} = 1.688\times10^{8}\ m/s$

c) V_g is max when $n - \lambda\frac{dn}{d\lambda}$ is min; since $\lambda\frac{dn}{d\lambda} < 0 \Rightarrow V_g$ is max when $\left|\lambda\frac{dn}{d\lambda}\right|$ is min

or $\Rightarrow \lambda \approx 1.3\mu m$



Problem 21 (Specialized: Optics-ECE4501) Code Number:________

Longitudinal Modes of Quantum Well Laser

Laser

$$\Delta\lambda_{gbw} = 6\,nm$$

$$\lambda = 1550\,nm \pm 3\,nm = 1547\,nm \text{ to } 1553\,nm$$

$$f = c/\lambda \quad \rightarrow \quad f_{min} = 193.0409\,THz \quad f_{max} = 193.7896\,THz$$

$$\Delta f_{gbw} = 0.7487\,THz$$

Resonant cavity

$$L = 250\,\mu m$$

$$n = 3.2771$$

Longitudinal mode spacing of cavity

$$\Delta f = \frac{c}{2\,n\,L} = 182.962\,GHz$$

Number of longitudinal modes at minimum and maximum of frequency range

$$f_{min}/\Delta f = 1055.087\,nm$$

$$f_{max}/\Delta f = 1059.179\,nm$$

Frequencies of the cavity

$$f = N\,\Delta f$$

and so

N	$f(THz)$	$\lambda(nm)$
1056	193.2079	1551.657
1057	193.3908	1550.189
1058	193.5738	1548.724
1059	193.7568	1547.262

are the modes within the lasing range.

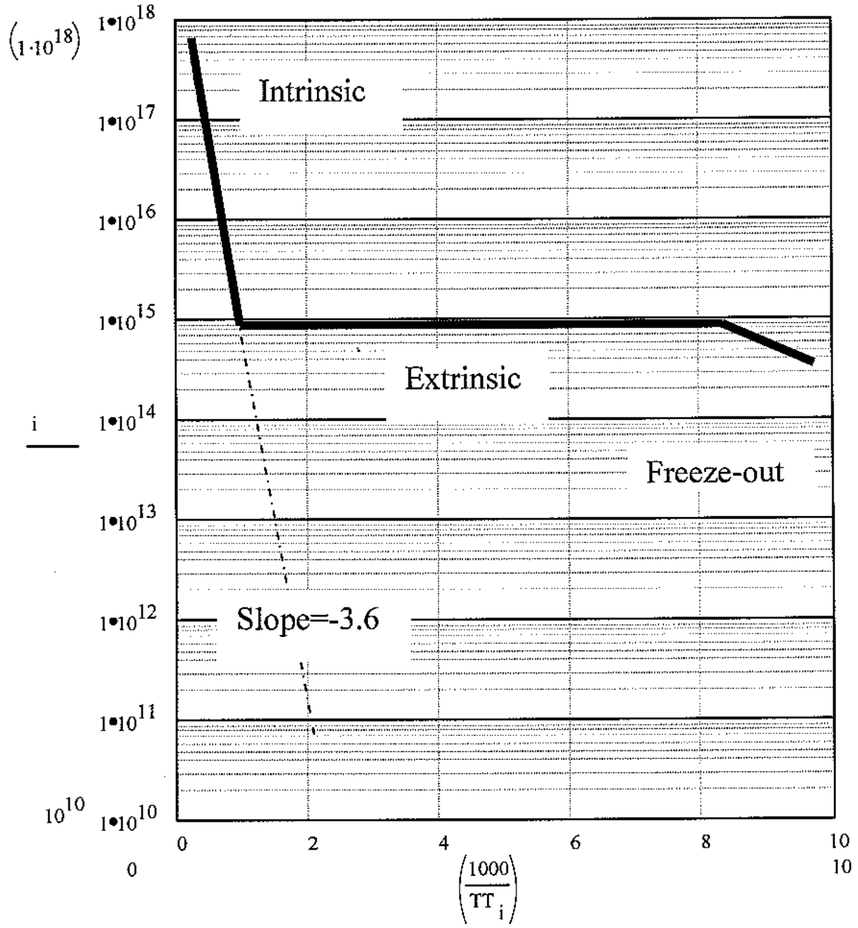
Problem 22 (Specialized: Microsystems-ECE4451 Code Number:

Solution

Carrier density statistics

a) Accurately plot the carrier density vs. inverse temperature for a GaAs (Nc=4.7x10^{17} cm^{-3}, Nv=9x10^{18} cm^{-3}) sample with a donor concentration of 10^{15}cm^{-3} and activation energy of 25 meV.

b) Identify the three primary regions of carrier density and explain the various regions of behavior.



We know GaAs E_g = 1.424 eV. Also the intrinsic carrier density is given by: $\eta_i = \sqrt{N_e N_v}\exp\left(\frac{-E_g}{2k_BT}\right)$ so the behavior in the intrinsic region is described by a straight line with a slope given by -Eg/(2k$_B$) (this slope is apparent on a natural log plot, the plot above is Log base 10 so a constant factor must be applied =Log(e). At very high temperatures the material is intrinsic. Very high is defined as when the carrier density is larger than the dopant density. Try T=1000. Then ηi=Sqrt(9*4.7)x10^{18} exp(-8.3) = 6.5x10^{18} x 0.248x10^{-3}= 1.6x10^{-15}cm^{-3} =y and x=1 is one point and the line has a slope given by -Eg/(2k) or the slope is -8.3x10^3 or -8.3xLog(e) =-8.3x.434= 3.6 for an x-axis units of 1000/T. Or you can simply calculate another point.

The extrinsic region is dominated by the ionization of the dopants and is described by a constant carrier density of 10^{15} cm^{-3}

The carrier freeze-out region occurs when the temperature (kT) is less than the activation energy i.e. when $kT<E_{activation}$

For Ea=10meV, the transition temp is near 300K(10meV/25.9meV)=115K or 1000/TT=8.7

This is seen directly in the expression for carrier density $n=\frac{N_D}{1+\frac{g_D n}{N_c}\exp\left(\frac{\Delta E_D}{kT}\right)}\approx\sqrt{\frac{N_c N_D}{g_D}}\exp\left(\frac{-\Delta E_D}{2kT}\right)$

The slope can be found in the same way as before and is 10meV/1.424=0.007 times 3.6

Problem 23 (Specialized: Bio Eng-ECE4781 Code Number:_________

A.(2 pts) Why do Action Potentials normally travel along a human axon in only one direction?

The Absolute Refractory Period makes the axon nonresponsive for a short period of time after each Action Potential firing. Hence the Action Potential cannot travel along the axon backwards.

B.(3 pts) What would happen to an Action Potential if the extracellular concentration of Potassium becomes 20% higher than normal?

A higher concentration of K+ outside the cell would reduce the K+ concentration gradient that pulls K+ ions out of the cell at the end of the Action Potential; therefore, a smaller number of K+ ions would leave the cell. The higher extracellular K+ concentration would also increase the electrical gradient that opposes the transport of K+ to the outside. The affect on Sodium transport would be more complicated. The initial electrical gradient pushing Na+ into the cell would be larger than normal; therefore the initial positive slope of the Action Potential curve would increase and the amount of Na+ entering the cell would also increase.

C.(2 pts) What would happen to the resting potential of the above neuron if the temperature is increased by a factor of 2 times the normal temperature?

The Nernst Equation claims that the Resting Membrane Potential would also double.

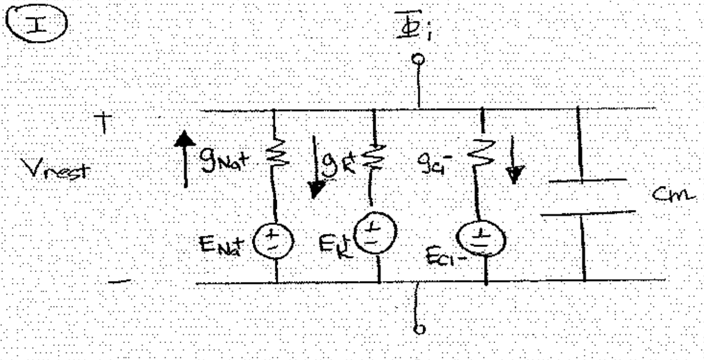
D.(3 pts) How is the Nerve Conduction Velocity measured for diagnoses of Carpal Tunnel Syndrome?

The nerve going to the hand is electrically stimulated and muscle potentials in the hand are measured to determine the delay caused by calcification of the foramen in the wrist. All of the above answers could be more quantitative.

Problem 24 (Specialized: Bio Eng-ECE4781) Code Number:__________

Parallel Conductance Model

(I)



(II)

$$V_{rest} = \frac{0.415(-64.54) + 0.01(+54.18) + 0.582(-65.77)}{1.007}$$

$$V_{rest} = -64.07\ mV$$

(III) Current Density Calculations

$$J_K = g_K(V_{rest} - E_K) = 0.415(-64.07 + 64.54)\frac{mS \cdot mV}{cm^2} = 0.195\frac{\mu A}{cm^2}$$

$$J_{Na} = g_{Na}(V_{rest} - E_{Na}) = 0.01(-64.07 - 54.18)\frac{mSmV}{cm^2} = -1.183\frac{\mu A}{cm^2}$$

$$J_{Cl} = g_{Cl}(V_{rest} - E_{Cl}) = 0.582(-64.07 + 65.77)\frac{\mu A}{cm^2} = 0.989\frac{\mu A}{cm^2}$$

(IV) During the upswing of an action potential the Na^+ conductance increase quickly thereby opening up (activating) Na^+ channels. As a result, Na^+ flows down the concentration gradient into the cell (fiber) and further depolarizes the membrane.

P. Bhatti

Problem 25 (Specialized: BioEng – ECE 4782) **Code Number: __________**

A. (4 points) Draw a box diagram of a model showing the large muscle groups (e.g. back muscles, arm muscles, etc) and sensory modalities (e.g. tactile cells on fingers, muscle spindle cells in back, etc) required for a healthy adult to lean over from a chair to tie a shoe.

The CNS box receives important sensory inputs from the retina, GTO's, muscle spindle cells, tactile cells on fingers and tactile cells on the butt. And don't forget that the cerebellum is involved with coordinated movements. The CNS controls the motor neurons which innervate skeletal muscle groups which then produces different "body mechanics," i.e. a separate box between each muscle group and the resulting motion. The important skeletal muscle groups are in the arms, hands, fingers, back and legs.

B. (3 points) Assume that all of the boxes in your model are linear and time invariant. Suggest experiments that would allow you to define the input-output characteristics of each box.

For each box, we would like to block all of the feedback paths that affect the box and control all of the inputs, e.g. providing a pulse stimulus to each box. The pulse input to tactile cells could be a pulse of pressure on the skin. The pulse input to GTO's and spindle cells could be a pulse of deformation. The stimulus to a muscle group could be a pulse of electrical current. The retina is way too complicated to characterize via a pulse stimulus, and it is also very nonlinear.

C. (3 points) Explain how you would "cut" some of the feedback loops when characterizing your model.

You could simply turn off the lights to block the retina feedback to the CNS. A topical anesthetic could block each group of tactile cells. Blocking the GTO's and muscle spindle cells would be more difficult, requiring an injection of an anesthetic near the ascending nerve fibers or physically cutting the nerve fibers.