Ph.D. Preliminary Examination Solutions
Spring 2010

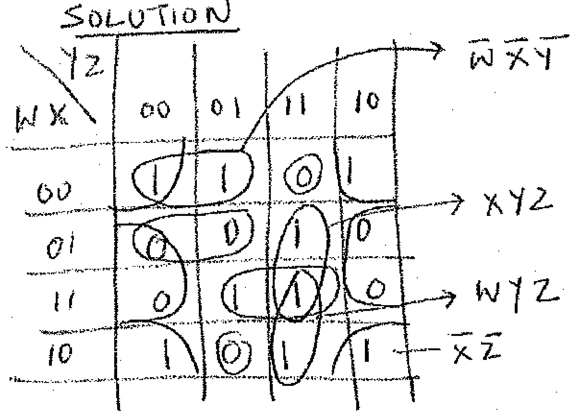
Instructions:

1. Please check to ensure that you have a complete exam booklet. There are 25 numbered problems. Note that **Problem 1 occupies 2 pages**, **Problem 2 occupies 2 pages,** and **Problem 18 occupies 2 pages**. Including the cover sheet, you should have **29 pages**. There should be no blank pages in the booklet.
2. The examination is closed book and closed notes. No reference material is allowed at your desk. A calculator is permitted.
3. All wireless devices must be turned off for the entire duration of the exam.
4. You may work a problem directly on the problem statement (if there is room) or on blank sheets of paper available from the exam proctor. Do not write on the back side of any sheet.
5. Your examination code number **MUST APPEAR ON EVERY SHEET**. This includes this cover sheet, the problem statement sheets, and any additional work sheets you turn in. **DO NOT** write your name on any of these sheets. Use the preprinted numbers whenever possible, or **WRITE LEGIBLY!!!**
6. Under the rules of the examination, you must choose 8 problems to be handed in for grading. Each problem to be graded should be separated from the rest of the materials, stapled to the associated worksheets, and placed on the top of the appropriate envelope in the front of the exam room. **DO NOT TURN IN ANY SHEETS FOR THE OTHER 17 PROBLEMS!!**
7. The examination lasts 4 hours, from 9:30 AM to 1:30 PM.
8. When you hand in the exam:
   (a) Separate the 8 problems to be graded as explained above.
   (b) Check to see that your Code Number is in **EVERY** sheet you are turning in.
   (c) On the section at the bottom of this page, **CIRCLE** the problem numbers that you are turning in for grading.
   (d) Turn in this cover sheet (containing your code number) and the 8 problems to be graded.
   (e) All other material is to be placed in the discard box at the front of the room. You are not allowed to take any of the exam booklet pages from the room!

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
19	**20**	**21**	**22**	**23**	**24**	**25**		

Problem 1 (Core: CompE-ECE2030) **Code Number:___________**

SOLUTION



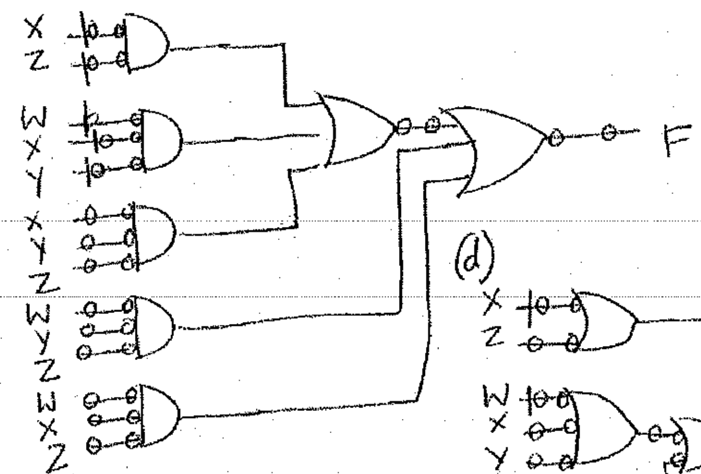
Consider the following K-map for the boolean function f( )

	00	01	11	10
00	1	1	0	1
01	0	0	1	0
11	0	1	1	0
10	1	0	1	1

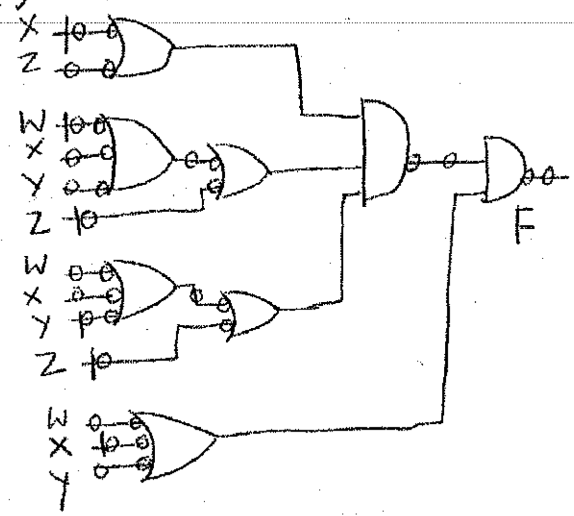
(a) $f = \overline{X}\overline{Z} + \overline{W}X\overline{Y} + XYZ + WYZ + \overline{W}XZ$

(b) $f = (\overline{X} + Z) \cdot (\overline{W} + X + Y + \overline{Z}) \cdot (W + X + \overline{Y} + \overline{Z}) \cdot (W + \overline{X} + Y)$

(c)

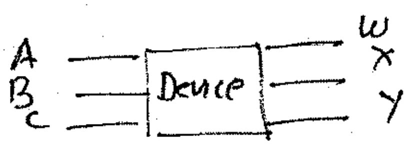


(d)



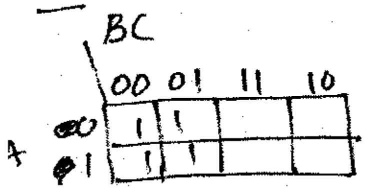
Problem 2 (Core: CompE-ECE2030) Code Number:__________

a. (4 pts) Consider an encryption device where a 3-bit input number is encrypted by performing a bit by bit exclusive-OR operation with a secret key 110 followed by a circular left shift of the result by one bit. Design and show a minimized circuit implementation of this device using only 2 input NAND gates.



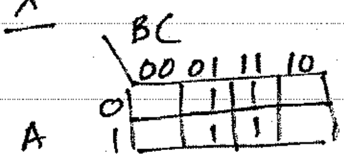
A B C	W X Y
0 0 0	1 0 1
0 0 1	1 1 1
0 1 0	0 0 1
0 1 1	0 1 1
1 0 0	1 0 0
1 0 1	1 1 0
1 1 0	0 0 0
1 1 1	0 1 0

W



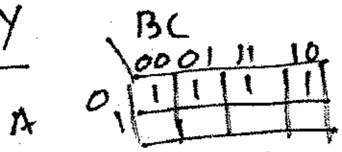
$W = \overline{B}$

X

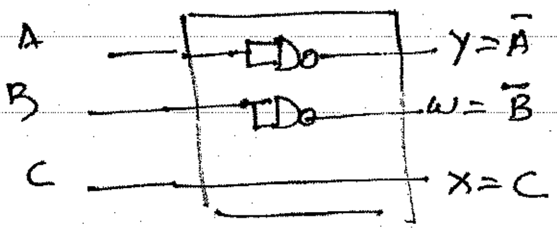


$X = C$

Y

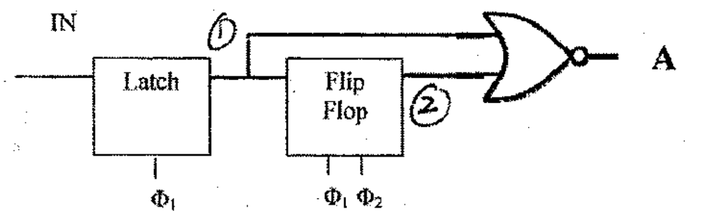


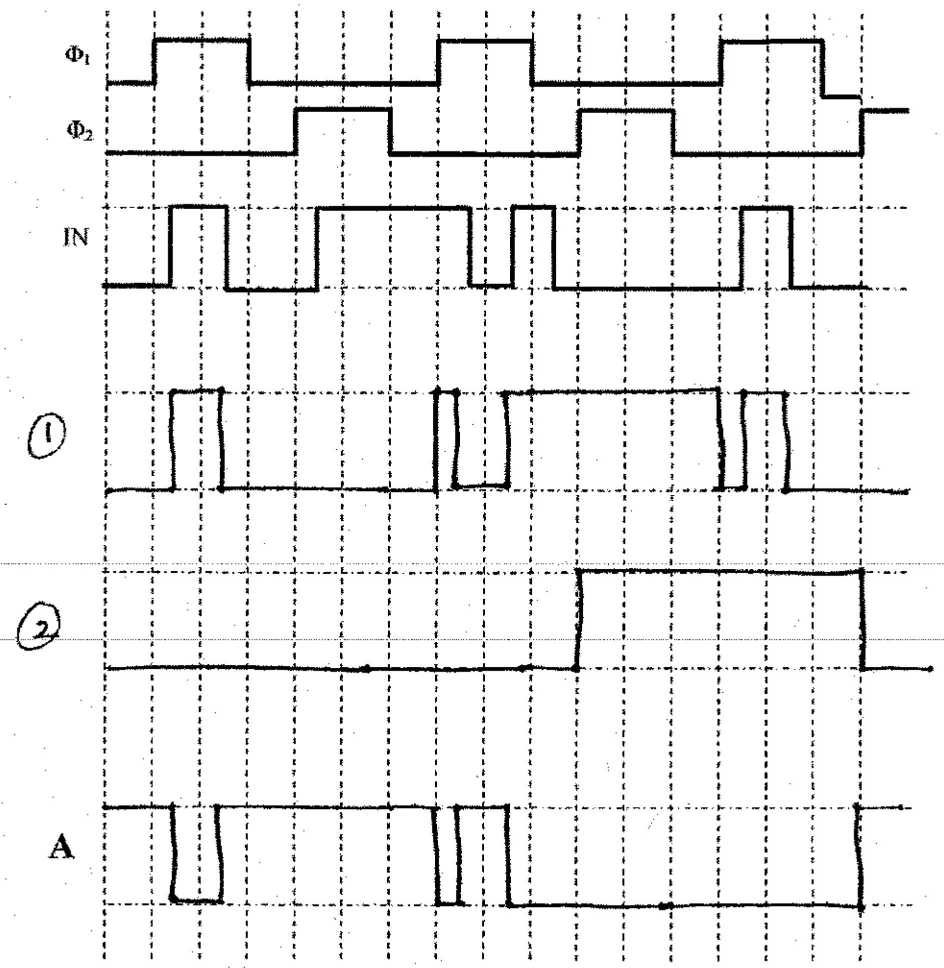
$Y = \overline{A}$



Problem 2 (Core: CompE-ECE2030) Code Number:__________

b. (6 pts) Fill in the timing of A. Flip flops use two phase non-overlapping clocks. Assume all registers and latches are initialized to 0 and ignore gate delays. State any other assumptions you wish to make.





Problem 3 (Core: CompE-ECE3055) Code Number:__________

The following RISC assembly language program is executed on a 32-bit MIPS processor. Fill in the register values that will be present, after execution of this program. A summary of MIPS instructions is included at the bottom of the page – for anyone unfamiliar with the MIPS instruction set. Prior to execution of the program, memory location 0x04000 contains 0x30552031. *Note:* 0x indicates hexadecimal and all answers must be in hexadecimal, default is decimal in the MIPS assembly language source file. A MIPS memory word or register contains 32-bits. Use XXXXXXXX for an undefined value.

```
         lw     $3, 0x04000
         srl    $4, $3, 9
         sub    $2, $4, $3
         xor    $3, $4, $2
         lui    $5, 10
         ori    $5, $5, 12562
         add    $6, $4, $3
         bne    $5, $6, LABEL1
         addi   $6, $0, -2031
LABEL1:  sw     $6, 0x04000
```

After execution of the MIPS code sequence above,

R2 = 0x___*CFC30A5F*________________(*in hexadecimal*)

R3 = 0x___*CFDB20CF*________________(*in hexadecimal*)

R4 = 0x___*00182A90*________________(*in hexadecimal*)

R5 = 0x____*000A3112*________________(*in hexadecimal*)

Memory Location 0x04000 contains: 0x____*CFF34B5F*_____________(*in hexadecimal*)

The MIPS processor contains thirty-two 32-bit registers, $0 through $31. $0 always contains a zero. By default, all arithmetic operations use two's complement arithmetic. Assume no branch delay slot is present.

MIPS Instruction			***Meaning***
ADDI	Rd, Rs, *Immed*	-	Rd = Rs + *Immediate* value
ADD	Rd, Rs, Rt	-	Rd = Rs + Rt (R – register ($))
ORI	Rd, Rs, *Immed*	-	Rd = Rs low 16-bits bitwise logical OR *Immediate* value
LUI	Rd, *Immed*	-	Rd = 16-bit *Immediate* value high 16-bits, 0's low 16-bits
BNE	Rs, Rt, *address*	-	Branch to *address*, only if Rs not equal to Rt
LW	Rd, *address*	-	LOAD - Rd gets contents of memory at *address*
SRL	Rd, Rs, *count*	-	Shift right logical (*use 0 fill*) by *count* bits
SUB	Rd, Rs, Rt	-	Rd = Rs - Rt
SW	Rd, *address*	-	STORE - memory at *address* gets contents of Rd
XOR	Rd, Rs, Rt	-	Rd = Rs bitwise logical XOR Rt

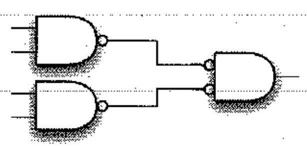
Problem 4 (Core: CompE-ECE3060)   Code Number:__________

You are part of a design team working on a new 3D integrated massively multi-core processor with 2^{18} cores. The implementation technology is CMOS. You are investigating the design of a hardware barrier synchronization mechanism in which the global AND ($B=\prod_{i=0}^{2^{18}-1} b_i$) of a signal b_i from each processor is computed and then distributed back to each processor. Assume a design style in which the ratio of pfet to nfet width in an inverter is $\gamma = 1$ and all gates are designed so that worst case risetime/falltime ratios correspond to that ratio for the inverter. Calculate delay in units of τ (the delay of an inverter driving an identically sized inverter), and include gate parasitic delays but neglect interconnect parasitic capacitance. Assume that the input capacitance of a minimum size inverter is C_{inv} and the parasitic delay of an inverter is τ. You may wish to solve the problem using the method of logical effort.

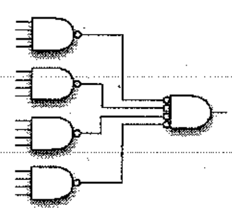
(a) Estimate the minimum delay required to compute B. Assume alternating k-input NAND and NOR gates (a radix k tree), and compare implementations with $k=2$ and $k=4$. Also assume that B drives a load of C_{inv}.

We'll use the method of logical effort. Then for $k=2$, since both NAND2 and NOR2 have logical effort $g=3/2$ when $\gamma=1$, and $H=1$ implies that all of the $h_i=1$, and thus the effort delay through each of the 18 stages is $f_i = g_i h_i = 1.5\tau$. Then including parasitic delay, the total delay through each stage is $d_i = f_i + p = 3.5\tau$, and since the total number of stages is 18, the total path delay is $D = 18 d_i = 63\tau$.

For $k=4$, $g=5/2$, and $f_i = g_i h_i = 2.5\tau$. Since the parasitic delay of a NAND4 or NOR4 is 4τ, we have $d_i = f_i + p = 6.5\tau$. A radix 4 tree implies 9 stages and $D = 9 d_i = 58.5\tau$



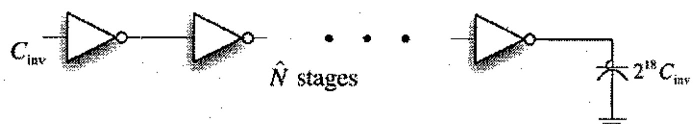
radix 2 stage



radix 4 stage

(b) Estimate the minimum delay to distribute B back to each processor.

Assume that each of the 2^{18} loads (one in each core) is C_{inv}. Then the path electrical effort $H = 2^{18}$, and path effort $F = GBH = 2^{18}$ ($G = B = 1$), and the ideal number of stages is $\hat{N} = \text{round}(\log_{3.6} F) = 10$. Then $\hat{f} = 2^{18/10} = 3.5\tau$ and $D = \hat{N}\hat{f} + P = 35 + 10 = 45\tau$



Problem 5 (Core: E&M-ECE3025) Code Number:__________

A coaxial transmission line is constructed from two conducting cylindrical shells, one of radius 2 mm and the other of radius 5 mm, centered along the z-axis and separated by a material with $\mu_r = 4.0$ and $\varepsilon_r = 1.0$. When energized so that a current flows down one cylinder and returns on the other, the magnetic field between the cylinders is

$$\overline{H} = \frac{7.958 \times 10^{-2}}{\rho}\hat{\phi} \quad \text{(A/m)}$$

where ρ is in units of meters. You may use $\mu_0 = 4\pi \times 10^{-7}$ H/m. Provide numerical answers whenever possible.

(a) What is the total current in Amperes on the inner conductor?

Solution: $I = \oint \overline{H} \bullet d\overline{\ell} = \int_0^{2\pi} \frac{7.958 \times 10^{-2}}{\rho} \rho d\phi = 0.5 \text{ A}$

(b) What is the surface current density $\overline{J}_S$ in A/m on the inner cylinder?

Solution: $\overline{J}_s\Big|_{\rho=0.002} = \hat{\rho} \times \overline{H}\Big|_{\rho=0.002} = \hat{z}\left(\frac{7.958 \times 10^{-2}}{0.002}\right) = \hat{z}39.79 \text{ A/m}$

(c) What is the magnetic flux density $\overline{B}$ as a function of ρ between the cylinders?

Solution: $\overline{B} = \mu_0 \mu_r \overline{H} = \frac{(4\pi \times 10^{-7})(4)(7.958 \times 10^{-2})}{\rho}\hat{\phi} = \frac{4.0 \times 10^{-7}}{\rho}\hat{\phi}$

(d) What is the total energy per unit length stored in these fields?

Solution:

$$W_m = \iiint \frac{1}{2}\mu \overline{H} \bullet \overline{H} = \int_0^1 \int_0^{2\pi} \int_{\rho=0.002}^{0.005} \frac{1}{2}\mu_0 \mu_r \left(\frac{7.958 \times 10^{-2}}{\rho}\right)^2 d\rho(\rho d\phi)dz$$

$$= \frac{2\pi}{2}(4\pi \times 10^{-7})(4)(7.958 \times 10^{-2})^2 \int_{\rho=0.002}^{0.005} \frac{1}{\rho} d\rho = (9.163 \times 10^{-8}) \text{ J/m}$$

(e) What is the total inductance L per unit length of the transmission line?

Solution: $W_m = \frac{1}{2}LI^2 \rightarrow L = \frac{2W_m}{I^2} = \frac{2(9.163 \times 10^{-8})}{(0.5)^2} = 7.33 \times 10^{-7} \text{ H/m}$

Problem 6 (Core: E&M-ECE3065) Code Number:__________

2. The electric field of a plane wave propagating in a nonmagnetic material is given by:

$\mathbf{E}=[\hat{y}\ 3\sin(2\pi\times 10^7 t - 0.4\pi x) + \hat{z}\ 4\cos(2\pi\times 10^7 t - 0.4\pi x)]$ (V/m)

(a) What is the direction of propagation? Is the medium lossy or lossless and why?

Propagation to $\hat{k} = +\hat{x}$ direction. The medium is lossless because there is no attenuation factor.

(b) Calculate the wavelength and the dielectric constant ϵ_r.

$k = 0.4\pi \Rightarrow \lambda = 2\pi/k = 2\pi/0.4\pi = 5\,m$

$u_p = \omega/k = 2\pi\times 10^7/0.4\pi = 5\times 10^7$ m/sec

$u_p = c/\sqrt{\epsilon_r\mu_r} \overset{\mu_r=1}{=\!=} \frac{c}{\sqrt{\epsilon_r}} \Rightarrow \epsilon_r = \left(\frac{c}{u_p}\right)^2 = 36$

(c) Calculate the intrinsic impedance of the medium η.

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r\mu_o}{\epsilon_r\epsilon_o}} = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_o}{\epsilon_o}} = \frac{120\pi}{6} = 20\pi\ (\Omega)$$

(d) Calculate the instantaneous expression of magnetic field **H**.

$$\bar{H} = \frac{1}{\eta}\hat{k}\times\bar{E} = \frac{1}{\eta}\hat{x}\times[\hat{y}\,3\sin(\ldots) + \hat{z}\,4\cos(\ldots)]$$

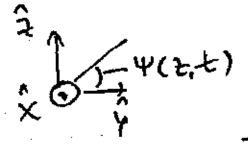
$$= \hat{z}\frac{3}{20\pi}\sin(2\pi 10^7 t - 0.4\pi x) - \hat{y}\frac{3}{20\pi}\cos(2\pi 10^7 t - 0.4\pi x)$$

(A/m)

(e) Does this wave feature linear, circular or eliptical polarization and why?

$|E_y| = 3,\ |E_z| = 4 \Rightarrow |E_y| \neq |E_z| \Rightarrow$ not circular

Modulus: $\psi(z,t) = \tan^{-1}\left(\frac{E_z}{E_y}\right) = \tan^{-1}\left(\frac{4}{3}\cot(\ldots t)\right) =$



= function of time t

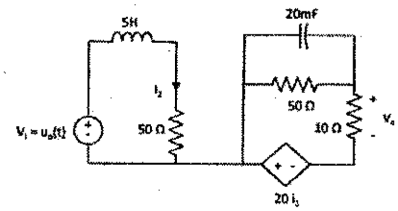
⇒ not linear

The polarization is elliptical

Problem 7 (Core: EDA-ECE2040) **Code Number:__________**

Solution: ECE 2040 Prelim Problem SPRING 2010

You may assume that all initial conditions that are needed to solve the following circuit are equal to zero. The input voltage is a unit step function.



A. What is the transfer function V_o/V_i in the Laplace domain?

$i_1 = \frac{V_i}{50+5s}$; using voltage division we know that $V_o = 20 i_1 \frac{10}{10 + 50 \| 1/s20m}$

$50 \| \frac{1}{s20m} = \frac{50}{1+(50)(20m)s} = \frac{50}{1+s}$; $V_o = \frac{20 V_i}{(50+5s)} \frac{10}{10 + 50/(1+s)}$

$$\frac{V_o}{V_i} = \frac{200}{5} \frac{(1+s)}{(s+10)(10+10s+50)}$$

$$\frac{V_o}{V_i} = \frac{40}{10} \frac{(1+s)}{(s+10)(s+6)}$$

$$\boxed{H(s) = \frac{V_o(s)}{V_i(s)} = 4 \frac{(1+s)}{(s+10)(s+6)}}$$

B. Is this circuit stable? Please justify your answer.

This circuit is stable because the poles of the transfer have a negative real part (i.e. in this case they are negative real numbers).

C. What is the output function, $V_o(t)$, as a function of time?

$V_i(s) = 1/s$; $V_o(s) = \frac{4}{5} \frac{(1+s)}{(s+10)(s+6)}$

Using a partial fraction expansion gives:

$$V_o(s) = \frac{4}{5} \frac{(1+s)}{(s+10)(s+6)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+6}$$

$$4+4s = A(s+10)(s+6) + Bs(s+6) + Cs(s+10)$$

$$4+4s = A(s^2+16s+60) + B(s^2+6s) + C(s^2+10s)$$

$$60A = 4 \therefore \boxed{A = 1/15}$$

$$\frac{16}{15} + 6B + 10C = 4$$

$$\frac{1}{15} + B + C = 0 \; ; \; B = -C - 1/15$$

$$6(-C - 1/15) + 10C = 4 - 16/15 = \frac{60-16}{15} = \frac{44}{15}$$

$$4C = 44/15 + 6/15 = 50/15 \; ; \; \boxed{C = 5/6}$$

$$B = -\frac{5}{6} - 1/15 = -\frac{25}{30} - \frac{2}{30} = \frac{-27}{30} \quad \boxed{B = -27/30}$$

Therefore;

$$V_o(s) = \frac{(1/15)}{s} - \frac{27/30}{s+10} + \frac{5/6}{s+6}$$

Knowing the inverse Laplace transform of $\frac{1}{s+a} = e^{-ta}$ allows us to easily get the inverse Laplace transform of $V_o(s)$ as

$$\boxed{V_o(t) = \left(1/15 - (27/30)e^{-10t} + (5/6)e^{-6t}\right) u_o(t)}$$

Problem 8 (Core: EDA-ECE3050) Code Number:__________

1. In the following circuit assume that the OpAmp is ideal.

a) Find the transfer function $A_v(s) = v_o(s)/v_s(s)$

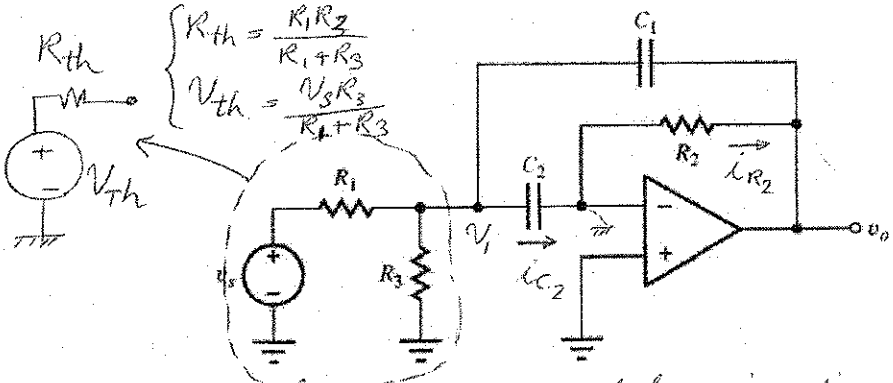
b) Draw the Bode plot (amplitude and phase).

c) What is the function of this circuit? Describe an application where it can be used.

d) Find the frequency at which the voltage gain is maximized if:

$$C_1 = C_2 = C,\quad R_3 \to \infty,\quad \omega_0 = \frac{1}{C\sqrt{R_1 R_2}},\ \text{and}\ Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

e) Find the bandwidth of this circuit (Hint: Q is the quality factor).



$$\begin{cases} R_{th} = \dfrac{R_1 R_2}{R_1 + R_3} \\ V_{th} = \dfrac{V_s R_3}{R_1 + R_3} \end{cases}$$

a) Negative terminal is virtually granded: $i_{C_2} = i_{R_2} = sC_2 V_1(s)$

$V_o(s) = -R_2 i_{R_2} = -R_2 sC_2 V_1(s) \Rightarrow V_1(s) = -\dfrac{V_o(s)}{sR_2C_2}$

KCL at node V_1: $\dfrac{V_1 - V_{th}}{R_{th}} + \dfrac{V_1 - V_o}{1/sC_1} + i_{C_2} = 0$

$\rightarrow \dfrac{V_{th}}{R_{th}} = -V_o(sC_1) + V_1\left(\dfrac{1}{R_{th}} + sC_1\right)$

$\dfrac{V_{th}}{R_{th}} = -V_o(s)\left(\dfrac{1}{sR_2R_{th}C_2} + \dfrac{C_1}{R_2C_2} + sC_1 + \dfrac{1}{R_2}\right)$

1

$$\Rightarrow \frac{V_o(s)}{V_{th}(s)} = \frac{-\frac{1}{R_{th}}}{\frac{C_1}{s}\left(\frac{1}{R_2 R_{th} C_1 C_2} + \frac{1}{R_2}\left(\frac{1}{C_2}+\frac{1}{C_1}\right)s + s^2\right)}$$

$$\Rightarrow \boxed{\frac{V_o(s)}{V_s(s)} = \frac{-\frac{s}{R_1 C_1}}{s^2 + \frac{s}{R_2}\left(\frac{1}{C_1}+\frac{1}{C_2}\right) + \frac{1}{R_2 R_{th} C_1 C_2}}}$$

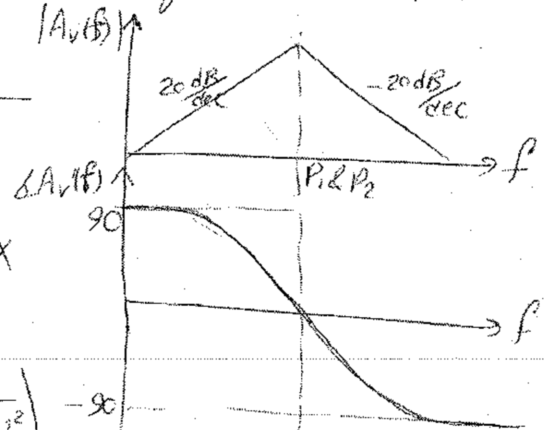
b) If we define: $\omega_o^2 = \frac{1}{R_2 R_{th} C_1 C_2}$ $\quad Q = \sqrt{\frac{R_2}{R_{th}}}\frac{\sqrt{C_1 C_2}}{C_1 + C_2}$

then: $\frac{V_o(s)}{V_s(s)} = -\sqrt{\frac{R_3}{R_1+R_3}\cdot\frac{R_2 C_2}{R_1 C_1}} \cdot \frac{s\omega_o}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2} = A_v(s)$

The Bode plot has one zero at the origin and two poles that have complex values.

c) Bandpass filter.

It can be used to select and amplify a certain frequency out of a complex wideband signal.



d) $\frac{\partial A_v(s)}{\partial s} = 0 = \frac{\partial}{\partial s}\left(\frac{\omega_o s}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}\right)$

$\rightarrow \left(s^2 + s\frac{\omega_o}{Q} + \omega_o^2\right) - s\left(2s + \frac{\omega_o}{Q}\right) = 0 \longrightarrow \boxed{s = \omega_o}$ Maximum $A_v(\omega)$ at $\omega_o = \omega$

e) $C_1 = C_2 = C \quad R_3 = \infty \rightarrow R_{th} = R_1$

$\Rightarrow \frac{V_o(s)}{V_s(s)} = -\sqrt{\frac{R_2}{R_1}} \frac{s\omega_o}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2} \quad \omega_o^2 = \frac{1}{R_2 R_1 C^2} \quad Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$

$Q = \frac{BW}{\omega_o} \Rightarrow BW = Q\omega_o = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \cdot \frac{1}{C\sqrt{R_1 R_2}} = \frac{1}{2R_1 C}$

Problem 9 (Core: Power-ECE3070) **Code Number:__________**

From OC test, we obtain the following values referred to the low voltage side:

$$R_C = \frac{(220)^2}{50} = 968\Omega$$

$$I_R = \frac{220}{968} = 0.227 \text{ A}$$

$$I_X = \sqrt{1^2 - (0.227)^2} = 0.9738 \text{ A}$$

$$X_m = \frac{220}{0.9738} = 225.9\Omega$$

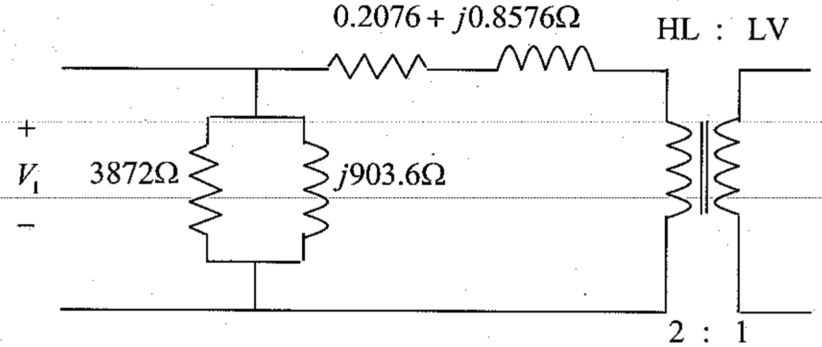
From the SC test, we obtain the following values referred to the high voltage side:

$$R_{eq} = \frac{60}{(17)^2} = 0.2076\Omega$$

$$Z_{eq} = \frac{15}{17} = 0.8824\Omega$$

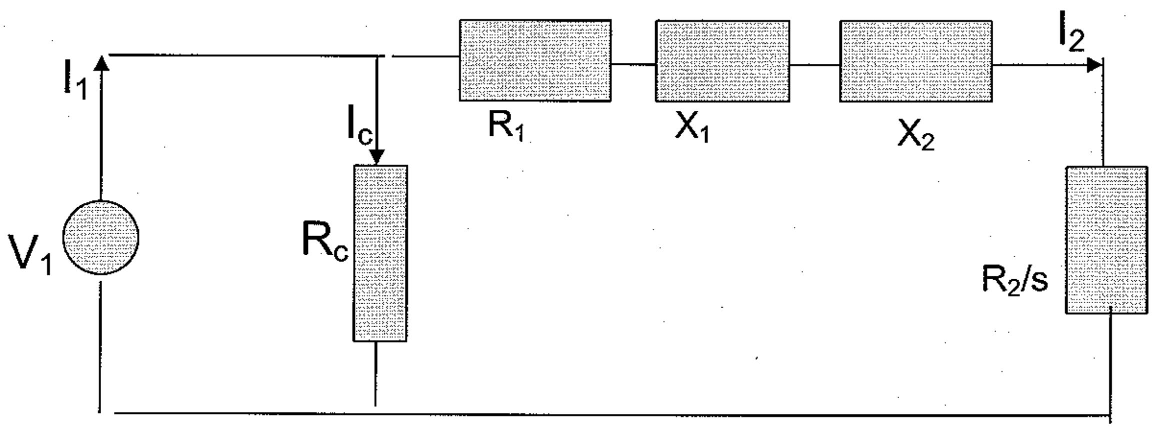
$$X_{eq} = \sqrt{(0.8824)^2 - (0.2076)^2} = 0.8576\Omega$$

Since the turns ratio a = 2.0, R_C and X_m' referred to the high voltage side are (968) 2^2 = 3872 Ω and $(225.9)2^2$ = 903.6 Ω, respectively. The equivalent circuit with all quantities referred to the high voltage side is:



Problem 10 (Core: Power-ECE3070) **Code Number:__________**

SOLUTION



$I_c = (230/1.732)/500 = 0.266$ ohm

Equivalent impedance of the series circuit is $Z_e = R_1 + R_2/s\ \ (jX_1\ \ + X_2\ ) = 10.5 + j1.25$

Hence equivalent rotor current is $I_2\ = (230/1.732)/(10.5 + j1.25) = 12.558 \angle - 6.79^o$

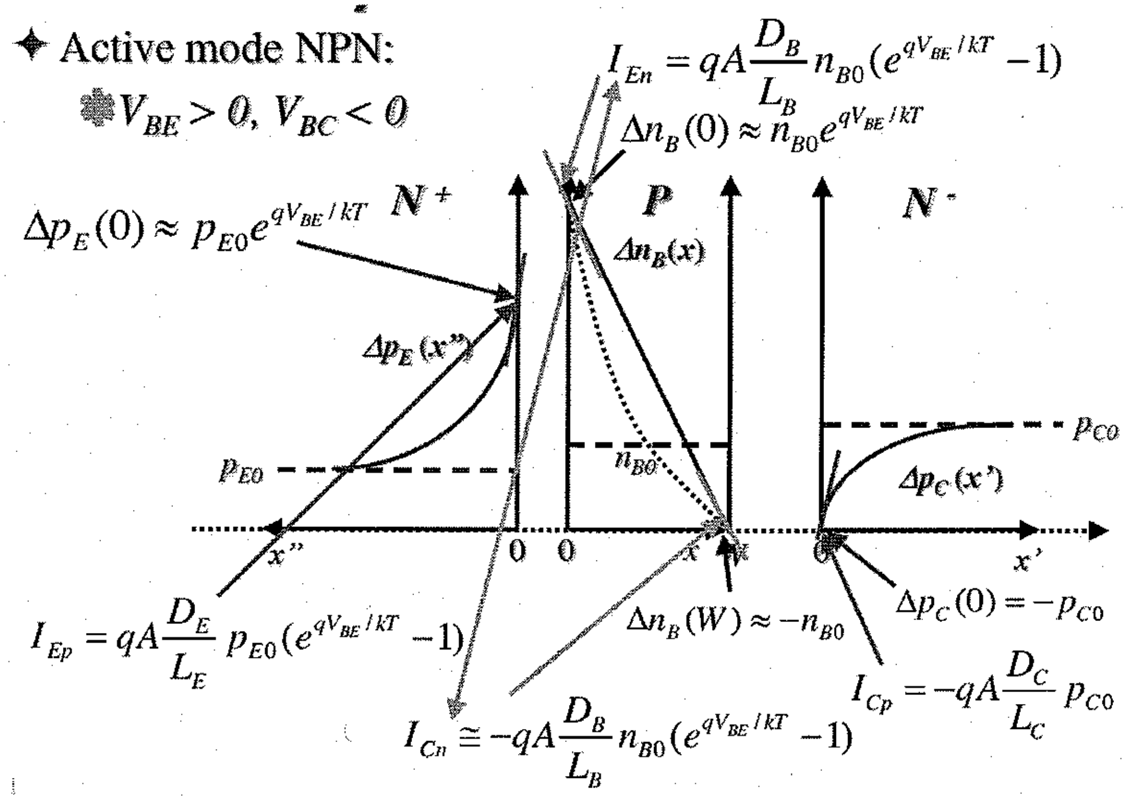
Stator and rotor copper losses are $= 3I_2^{\ 2}\ \ (0.5 + 0.25) = (3)12.558^2\ (0.75) = 344.84$ W

Core loss $= 3(230/1.732)^2 / 500 = 106.13$ W

Power output $= 3I_2^{\ 2} R_2 /s\ = \ (3)12.558^2\ (0.25/0.025) = 4462.82$ W

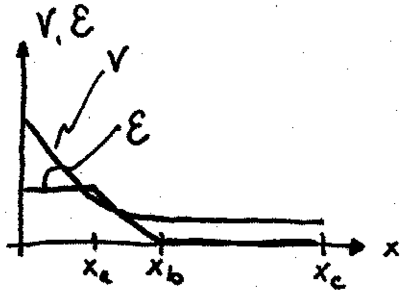
Efficiency = 4462.82 /(4462.82 +344.84 +106.13) = 87.96%

Problem 11 (Core: Microsystems-ECE3040) Code Number:__________

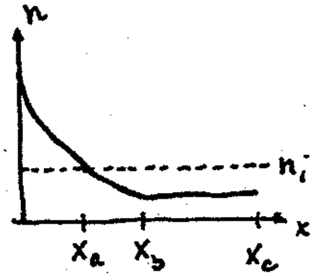


Problem 12 (Core: Microsystems-ECE3080) Code Number:__________
Solution

(a)



(b)



$x = x_a$: $E_F = E_i \Rightarrow n = n_i = 10^{10}\,cm^{-3}$

$x = x_c$: $E_i - E_F = E_g/4 \Rightarrow n = n_i\, e^{(E_F - E_i)/kT} = n_i\, e^{-E_g/4kT}$

$= 2.0 \cdot 10^5\,cm^{-3}$

(c) $x = x_a$: $n = p = n_i$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)} = \frac{1}{q n_i(\mu_n + \mu_p)} = 3.43\ 10^5\,\Omega cm$$

$x = x_c$: $n = n_i\, e^{-E_g/4kT} = 2 \cdot 10^5\,cm^{-3}$

$p = n_i\, e^{+E_g/4kT} = 5.0 \cdot 10^{14}\,cm^{-3}$

$$\rho \approx \frac{1}{q\mu_p p} = 27\,\Omega cm$$

(e)

$E_{kin} = E_c(x = x_b) - E(x=0)$

$= \frac{3}{4} E_g = 0.84\,eV$

(d) There is a diffusion current because there is a gradient in n, p
There is a drift current because there is an E-field
The total current must be zero, because E_F is constant.

(f) $\Delta n = \Delta p = G_L \tau_n = 10^{15}\,cm^{-3}$

$$\rho = \frac{1}{q[\mu_n(n+\Delta n) + \mu_p(p + \Delta p)]} = 3.06\,\Omega cm$$

Problem 13 (Core: DSP-ECE2025) Code Number:__________

PROBLEM SOLUTION:

(a) (5 pts) Suppose we have a **linear, time-invariant** continuous-time system, and that the system outputs the signal

$$y_1(t) = 8(t-5)^3 \cdot 3e^{-2(t-5)}u(t-5)$$

when it is given the input signal

$$x_1(t) = 4(t-4)^2 \cdot 2e^{-2(t-4)}u(t-4).$$

Find the output of the system, $y(t)$, if the input is

$$x_2(t) = 12(t-6)^2 \cdot 2e^{-2(t-6)}u(t-6) - 4(t-8)^2 \cdot 2e^{-2(t-8)}u(t-8)$$

Note: the signal $u(t)$ is the *unit-step* signal.

Solution: Note that $x_2(t) = 3x_1(t-2) - x_1(t-4)$. By linearity and time-invariance,

$$y_2(t) = 3y_1(t-2) - y_1(t-4) = 24(t-7)^3 \cdot 3e^{-2(t-7)}u(t-7) - 8(t-9)^3 \cdot 3e^{-2(t-9)}u(t-9)$$

(b) (5 pts) Suppose that we have a discrete-time system whose output $y[n]$, given an input $x[n]$, is specified by

$$y[n] = 4\sum_{k=5}^{15} x[n-k]$$

Consider the input $x[n] = 8\sin(\hat{\omega}n + \pi/7)$. For what value(s) of $\hat{\omega}$ does $y[n] = 0$ for all n? If there are many such frequencies, $\hat{\omega}$, then clearly and concisely specify all of them in the interval $0 \le \hat{\omega} < 2\pi$.

Solution: Most of the details in this problem are not necessary for obtaining the solution. The impulse response is a "boxcar". The frequency response is thus a "digital sinc", or Dirichlet function.

$$H(e^{j\hat{\omega}}) = e^{j\varphi(\hat{\omega})}\frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)}$$

In the frequency response, it is only the locations of the zeros of this function that are relevant. Time shifts do not change the location of the zeros, so a boxcar from -5 to $+5$ will do as well as a boxcar from 5 to 15. The length of the boxcar is eleven, i.e., $L = 11$. The Dirichlet has zeros at $\hat{\omega} = 2\pi k/11$ for $k \neq 0$, and $k = 1, 2, \ldots, 10$.

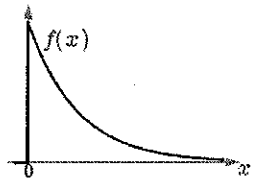
Thus the answer is:

$$\hat{\omega} = 2\pi k/11, \quad \text{for } k = 1,2,3,4,5,6,7,8,9,10$$

Problem 14 (Core: DSP-ECE3075) Code Number:__________

3075 SOLUTION

Let X and Y be independent identically distributed random variables with pdf $f(x) = e^{-x}u(x)$, as sketched to the right. (The unit step is $u(x) = 1$ for $x > 0$ and $u(x) = 0$ elsewhere.)



(a) **The mean and variance of $Z = 2X - 3Y$ are $E(Z) =$** [−1] **and $\text{var}(Z) =$** [13].

The mean is $E(Z) = E(2X - 3Y) = 2E(X) - 3E(Y) = 2 - 3 = -1$.

The variance is $\text{var}(Z) = \text{var}(2X - 3Y) = 2^2\text{var}(X) + (-3)^2\text{var}(Y) = 4 + 9 = 13$.

(b) **Find $E(e^{0.9(X+Y)}) =$** [100].

But X and Y are independent

$\Rightarrow$ this reduces to $E(e^{0.9X})E(e^{0.9Y})$, where both factors are identical:

$$E(e^{0.9X}) = \int_{-\infty}^{\infty} e^{0.9x} f(x)dx$$

$$= \int_0^{\infty} e^{0.9x}e^{-x}dx = \int_0^{\infty} e^{-0.1x}dx = 10.$$

(c) **The pdf for the ratio $R = \dfrac{X}{Y}$ is $f(r) =$** $\boxed{\dfrac{1}{(1+r)^2}u(r)}$.

The ratio is never negative, so the cdf has a $u(r)$ factor in it.

For positive r, the cdf for R is:

$$F(r) = P(R \le r)$$

$$= P(\frac{X}{Y} \le r)$$

$$= P(X \le rY)$$

$$= E_Y(P(X \le rY | Y)) \qquad \text{(Law of Total Expectation)}$$

$$= E(F_X(rY))$$

$$= E(1 - e^{-rY}))$$

$$= 1 - \frac{1}{1+r} \qquad \Rightarrow \qquad f(r) = \frac{d}{dr}F(r)$$

$$= \frac{1}{(1+r)^2}u(r).$$

Problem 15 (Core: S&C-ECE3085) Code Number:__________

Solution

i) Note that $h_N(t) \geq 0$, since each term is.

$$\int_0^\infty |h_N(t)|\,\mathrm{d}t = \sum_{i=0}^{N-1} \int_0^\infty t^{2k} \sin^{2k} t\,\mathrm{d}t.$$

These integrals diverge, hence the system is not BIBO stable.

ii) Express now $h_N(t)$ in terms of harmonics.

$$\begin{aligned}
\sin^{2k} t &= \left(\frac{\mathrm{e}^{jt} - \mathrm{e}^{-jt}}{2j}\right)^{2k} \\
&= \frac{1}{(2j)^{2k}} \sum_{i=0}^{2k} \binom{2k}{i} \mathrm{e}^{jit} \left(-\mathrm{e}^{-jt}\right)^{2k-i} \\
&= \frac{1}{4^k} \sum_{i=0}^{2k} \binom{2k}{i} (-1)^{i+1} \cos 2(k-i)t.
\end{aligned}$$

This shows that $\sin^{2k} t$ is a linear combination of $1, \cos 2t, \cos 4t, \ldots, \cos 2(k-1)t, \cos 2kt$. Thus $\sin^{2(N-1)} t$ is nulled by the differential operator

$$\mathbf{D}(\mathbf{D}^2 + 2^2)(\mathbf{D}^2 + 4^2) \cdots (\mathbf{D}^2 + (2(N-1))^2) = \mathbf{D} \prod_{i=1}^{N-1} (\mathbf{D}^2 + (2i)^2).$$

Consequently, $t^{2(N-1)} \sin^{2(N-1)}$ is nulled by the differential operator

$$\left(\mathbf{D} \prod_{i=1}^{N-1} (\mathbf{D}^2 + (2i)^2)\right)^{2N-1}$$

Note that this operator also nulls all lower powers $t^{2i} \sin^{2i}$ for $i = 1, \ldots, (N-1)$. Hence these do not bring any additional factors to the differential operator. We conclude that $h_N(t)$ satisfies the differential equation

$$\left(\mathbf{D} \prod_{i=1}^{N-1} (\mathbf{D}^2 + (2i)^2)\right)^{2N-1} h_N = 0.$$

Its order is $(2N-1)^2$.

iii) The general solution to the ODE

$$\left(\mathbf{D}\prod_{i=1}^{N-1}(\mathbf{D}^2+(2i)^2)\right)^{2N-1} x = 0.$$

is

$$x(t) = P_0(t) + \sum_{i=1}^{N-1}\left(P_i(t)\cos 2it + Q_i(t)\sin 2it\right)$$

where the $P_i(t)$ and $Q_i(t)$ are arbitrary polynomials of degree $2(N-1)$.

iv) From (iii), we see that all periodic solutions are of the form

$$x_p(t) = \sum_{i=1}^{N-1}\left(P_i\cos 2it + Q_i\sin 2it\right)$$

where now P_i and Q_i are constants.
The minimal and maximal frequency are respectively $1/\pi$ and $(N-1)/\pi$ Hz.

Problem 16 (Core: S&C-ECE3085) **Code Number:__________**

Solution

(a) $E(s) = R(s) - Y(s)$

$Y(s) = E(s)G_c(s)G_p(s) + D(s)G_p(s)$

$E(s) = R(s) - E(s)G_c(s)G_p(s) - D(s)G_p(s)$

$$E(s) = \frac{R(s) - D(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

(b) $$E(s) = \frac{R(s) - D(s)\frac{3}{s-7}}{1 + \frac{K_p s + K_i}{s}\cdot\frac{3}{s-7}} = \frac{s(s-7)\left[R(s) - \frac{3D(s)}{s-7}\right]}{s^2 - 7s + 3K_p s + 3K_i}$$

$$E(s) = \frac{s(s-7)R(s) - 3sD(s)}{s^2 + (3K_p - 7)s + 3K_i}$$

(c) $R(s) = \frac{A}{s}$, $D(s) = \frac{B}{s}$

$$E(s) = \frac{s(s-7)\frac{A}{s} - 3s\frac{B}{s}}{s^2 + (3K_p - 7)s + 3K_i}$$

$$= \frac{A(s-7) - 3B}{s^2 + (3K_p - 7)s + 3K_i}$$

$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$ if the limit exists

For the limit to exist, $e(t)$ has to be stable.

Thus, $K_p > 7/3$ and $K_i > 0$

(d)

$$G_{cl}(s) = \frac{\frac{3(K_p s + K_i)}{s(s-7)}}{1 + \frac{3(K_p s + K_i)}{s(s-7)}} = \frac{3K_p s + 3K_i}{s^2 + (3K_p - 7)s + 3K_i}$$

Poles at $-4 \pm j \Rightarrow$ denominator $= (s+4)^2 + 1^2 = s^2 + 8s + 17$

Equate coefficients:

$3K_p - 7 = 8$ $\quad$ $3K_i = 17$ $\quad$ $G_c(s) = 5 + \frac{17/3}{s}$

$K_p = 5$ $\quad$ $K_i = 17/3$

$$G_{cl}(s) = \frac{15s + 17}{s^2 + 8s + 17}$$

$r(t) = u(t)$, $\quad R(s) = \frac{1}{s}$; $\quad Y(s) = R(s)G_{cl}(s) = \frac{15s+17}{s(s^2+8s+17)}$

$y_{ss} = \lim_{s \to 0} sY(s) = 1 \Rightarrow e_{ss} = 0$ (zero steady state error)

(e) $Y(s) = -G_c(s)G_p(s)Y(s) + D(s)G_p(s)$

$$Y(s) = \frac{D(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

$$H_d(s) = \frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + G_c(s)G_p(s)} = \frac{\frac{3}{s-7}}{1 + \left(\frac{5s + 17/3}{s}\right)\left(\frac{3}{s-7}\right)}$$

$$= \frac{3s}{s^2 - 7s + 15s + 17} = \frac{3s}{s^2 + 8s + 17}$$

$$d_{ss} = \lim_{s \to 0} sD(s) = \lim_{s \to 0} s\left(\frac{1}{s}\right)\frac{3s}{s^2+8s+17} = 0$$

(f) In the steady state, this overall system tracks a step input with zero error while completely rejecting a step disturbance.

Problem 17 (Specialized: Comp Science-CS3210) Code Number:______

Solution to Barrier Problem

The reason BuggyBarrier does not work is as follows. The `CountBarrier` variable does in fact count up to `numprocs` once all threads have entered the barrier. The last thread entering then clears the `CountBarrier` variable to zero, informing all other threads it is time to leave the barrier and continue the computation. This is the correct behavior. The problem occurs wnen one or more threads exit the barrier (after observing `CountBarrier` equal to zero), and *then re–enters the barrier for the next iteration* before one or more threads from the previous barrier have observed the `CountBarrier` variable decremented to zero. At this point, `CountBarrier` is non–zero, due to the `FetchAndIncrement`, and the threads from the prior barrier entry believe they are still waiting for all thread to enter previously.

Algorithm BuggyBarrier

This Can't Possibly Work!

```
int CountBarrier = 0; /* Global Variable */
void BuggyBarrier() {:
  mycount = FetchAndIncrement(&CountBarrier); /* Atomic */
  if(mycount == (numprocs - 1)) {
    CountBarrier = 0; // All there, let others know and reset
  else
    while(CountBarrier != 0) spin() // Wait for others
}
```

Problem 18 (Specialized: Software Sys- ECE3035) Code Number: _____

Part 1: The following figure shows the content of the stack when 'main' is ready to call 'foo' (suppose the stack is at 6000 when 'foo' is called.) Fill the content of the stack when 'foo' finishes the first 'for' loop.

address	content
...	...
6000	6 (the input parameter)
5996	1000 the return address (of 'x=x+1')
5992	**4 a[4]**
5988	**3**
5984	**2**
5980	**1**
5976	**0 a[0]**
5972	**5 i**

Use more rows as necessary

Part 2: Fill the content of the stack when 'foo' finishes the second 'for' loop and is about the return.

address	content
...	...
6000	6
5996	**996 modification of the return address due to buffer overflow**
5992	**0 a[4]**
5988	**-1**
5984	**-2**
5980	**-3**
5976	**-4 a[0]**
5972	**6 i**

Use more rows as necessary

Part 3: What will happen to the code?

Because 996 is the address of "jal foo", the code will execute an infinite loop, calling function foo again and again.

Problem 19 (Specialized: Telecom-ECE3076) Code Number:__________

What are the characteristics of an Internet Autonomous System (AS).

1. It has a block of Internet Protocol (fill in the word) _______**addresses**__________ ,

2. assigned by what organization? __________**IANA** (or Internet Assigned Numbers Authority)_____

3. If it also a "domain", it must maintain two servers, of what type? ______**DNS (or Name Servers)**________

4. An Autonomous System has a boundary router or routers that connect to the worldwide Internet using what routing protocol? ________**BGP** (or Border Gateway Protocol)____________

5. What routing protocol is most likely to be used internally by a contiguous AS like Georgia Tech?

   ____**OSPF** (or Open Shortest Path First)_____

---

6. What causes the transit time for Internet datagrams to vary? ______**Heavy Traffic** causes buffers to fill________

7. What causes Internet datagrams to arrive out of order? ____**Varying delays cause forwarding paths to shift**____

8. Why are there no MAC address conflicts when a local area network is a mixture of wired Ethernet (IEEE 802.3) hosts and wireless WiFi (IEEE 802.11) hosts.

   ___**Addresses for Ethernet and WiFi come from the same pool** (uniquely assigned by IEEE to manufacturers)__

9. If signals travel at 2E8 m/s through 100 Mb/s fiber and wire connections, what is the minimum round trip time, **RTT**, between hosts over a 4000 km path length (neglecting any delay at nodes)?

   ______________RTT > 2 * 4e6 m / 2e8 m/s = **0.040 seconds**______________

---

10. What is the maximum data transfer rate for a single TCP connection between a host in Atlanta and a host in Chattanooga if the round trip delay is 20 milliseconds, the connection bit rate is 100 Mbps, and the TCP window size is 24,000 bytes?

   ______Window Limit = 8 bits/byte * 24e3 bytes / 0.020 s =9,600,000 b/s = **9.6 Mb/s**________

   ______________(since 9.6 Mb/s < 100 Mb/s)

Problem 20 (Specialized: Optics-ECE4500) Code Number:__________

Focused Laser Beam Spot Size

$$\lambda = 488.0\, nm$$

$$D' = 2.0\, mm \;\; \text{or} \;\; 5.0\, mm$$

$$f = 25.0\, mm \;\; \text{or} \;\; 50.0\, mm$$

Full angle of beam divergence/convergence

$$\theta = 2\, tan^{-1} \frac{\lambda}{\pi\, (D/2)}$$

$$tan\, \frac{\theta}{2} = \frac{2}{\pi} \frac{\lambda}{D}$$

Diffraction-limited focusing of beam of diameter D'

$$tan\, \frac{\theta}{2} = \frac{D'/2}{f} = \frac{D'}{2\, f}$$

Equating

$$\frac{2}{\pi} \frac{\lambda}{D} = \frac{D'}{2\, f}$$

$$D = \frac{4}{\pi} \frac{\lambda\, f}{D'}$$

Therefore choose $D' = 5.0\, mm$ and $f = 25\, mm$.

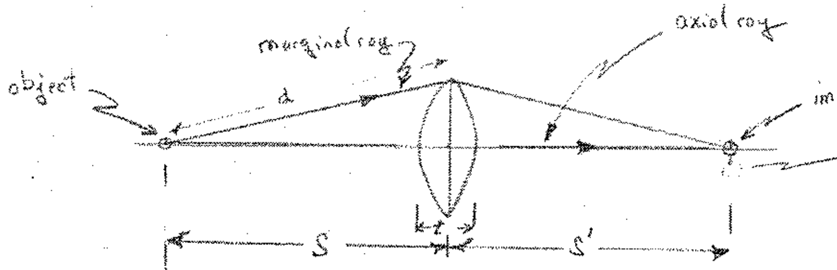
$$D = 3.1067\, \mu m$$

Problem 21 (Specialized: Optics-ECE4501) Code Number:__________

Geometric Optics

A symmetric double convex lens is circular, having a diameter of 5cm and zero thickness at its edge. A point object on the axis through the center of the lens (optic axis) produces a real image on the opposite side. Both object and image distances are 30 cm, measured from the plane, normal to the axis, that bisects the lens. The lens refractive index is 1.520.

a) Sketch the lens together with two paths; one path being a marginal ray and the other being an axial ray, identify each.
b) What can be said of the two optical paths (marginal ray path and axial ray path) between object and image?
c) Determine the thickness of the lens on-axis.
d) What is the focal length of the lens?
e) If the point object is moved 1mm above the optic axis where is the new image located?



S = 30 cm
S' = 30 cm
t = lens thickness on axis

b) marginal ray optical path = axial ray optical path (from Fermat's principle)
optical path = index of refraction × geometric pathlength

c) $d = \frac{1}{2}$ marginal ray optical path $= \sqrt{30^2 + (D/2)^2}$ D = lens diameter
$= 30.104$ cm

axial ray path $= S + S - t + nt = 30.0 + 30.0 - t + 1.520\,t$
$= 60.0 + t(.520)$

$\therefore \quad 2 \times 30.104 = 60.208 = 60.0 + t(.520)$

$t = .40$ cm $= 4$ mm

d) lens eq: $1/f = 1/S + 1/S'$
$= 2/30.0\text{ cm} \Rightarrow f = 15$ cm

e) 30 cm along the image axis and 1mm below } neglects spherical aberration

Problem 22 (Specialized: Microsystems-ECE4752) Code Number:_____

(a) **During the predeposition, the source is on all the time, therefore, surface concentration is equal to the solubility.**

Thus N_s=1.2x10^{20}

Dose during the predeposition is given by

$$Q = 2N_S\sqrt{\frac{D_1 t_1}{\pi}}$$

$$= 2\times 1.2\times 10^{20}\sqrt{\frac{1.45\times 10^{-15}\cdot 900}{\pi}}$$

$$= 1.42\times 10^{14}\,\mathrm{cm}^{-2}$$

(b) **For a two step diffusion process, effective $(Dt)_{eff}=D_1t_1+D_2t_2$**

$$\begin{aligned}(Dt)_{eff} &= D_1 t_1 + D_2 t_2\\ &= 1.45\times 10^{-15}\cdot 900 + 2.96\times 10^{-13}\cdot 18000\\ &= 1.31\times 10^{-12} + 5.33\times 10^{-9}\\ &= 5.33\times 10^{-9}\ \mathrm{cm}^2\end{aligned}$$

Junction depth after drive-in

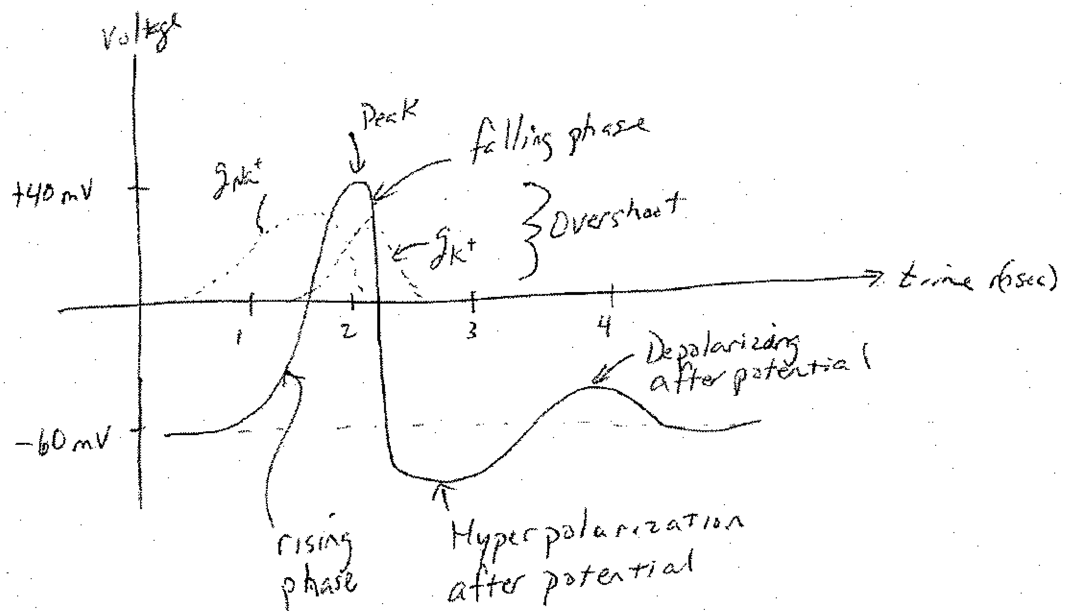
$$x_j = 2\sqrt{(Dt)_{eff}\cdot \ln\left(\frac{N_S}{N_B}\right)}$$

$$= 2\sqrt{5.33\times 10^{-9}\cdot \ln\left(\frac{1.1\times 10^{18}}{3\times 10^{16}}\right)}$$

$$= 2.77\mu\mathrm{m}$$

Problem 23 (Specialized: Bio Eng-ECE4784) Code Number:__________

Draw a general action potential for a nerve. Include the temporal characteristics for the sodium and potassium conductances. Include relative numbers for the potential and time axes. Label the schematic with critical features, regions, values, etc.



Problem 24 (Specialized: Bio Eng-ECE4782) Code Number:__________

The intracellular and extracellular concentrations and conductances for the axon of a newly discovered squid species at rest are given below.

A) Find the Nernst potential for Na^+, K^+, and Cl^-.

B) Find the membrane resting potential.

Species	Intracellular (mM)	Extracellular (mM)	Conductances (mS/cm^2)
K	500	10	0.415
Na	70	350	0.010
Cl	24	350	0.582

A) $E_{Na^+} = -\frac{KT}{q} \ln\left(\frac{70}{350}\right) = 43.0 \text{ mV}$ ($\frac{KT}{q} \to 0.0267$; ~~37.6~~)

$E_{K^+} = -\frac{KT}{q} \ln\left(\frac{500}{10}\right) = -104.5 \text{ mV}$

$E_{Cl^-} = -\frac{KT}{q} \ln\left(\frac{24}{350}\right) = -71.6 \text{ mV}$

B) $$V_{rest} = \frac{g_K E_K + g_{Na} E_{Na} + g_{Cl} E_{Cl}}{g_K + g_{Na} + g_{Cl}}$$

$$= \frac{(-104.5)(0.415) + (43.0)(0.010) + (-71.6)(0.582)}{0.415 + 0.010 + 0.582} \text{ mV}$$

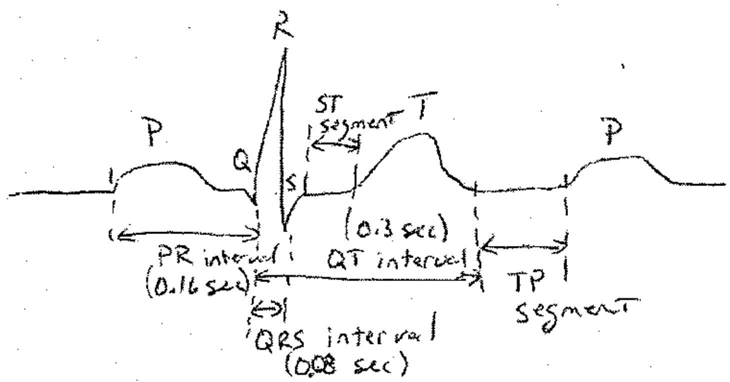
$V_{Rest} =$ ~~-[illegible]~~ -84.6 mV

Problem 25 (Specialized: Bio Eng-ECE4781) Code Number:__________

Draw a standard electrocardiogram for a healthy human being. Label the intervals and describe the relevant heart function for each interval.

The resting (or filling) phase of the heart cycle is called the ___diastole____.

The contractile (or pumping) phase of the heart cycle is called the ___systole____.



PR interval - measure of the AV conduction time
QRS complex – activation of the ventricals
TP segment – establishes a baseline for the measurement
T wave – recovery of the ventrical cardiac cells
QT interval – total duration of the ventricular systole